K(II)-Computer Science-H-3

2018

COMPUTER SCIENCE – HONOURS

Third Paper

Full Marks : 100

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any five*** from the rest, taking at least ***two*** from Group A and at least ***one*** each from Group B and Group C.

1. Answer ***any ten*** questions. 2×10

(a) If $y = 4x^6 - 5x$, find the percentage error in y at x = 1, if the error in x = 0·04.

(b) How many seven-letter words can be formed using the letters of the word 'BENZENE'?

(c) Define the Graph Colouring problem.

(d) A couple has two children. Find the probability *p* that both children are boys if it is known that (i) at least one of the children is a boy, (ii) the older child is a boy.

(e) When is a graph said to be a regular graph? Draw a regular graph of degree 3 having more than 4 vertices.

(f) What is an initial value problem and a boundary value problem?

(g) Five cards are numbered 1 to 5. Two cards are drawn at random. Let x denote the sum of the numbers drawn.

(i) Find the distribution of x.

(ii) Find the mean μ, variance σ^2 = Var (x) and the standard deviation $\sigma = \sigma_x$ of x.

(h) 'Every edge of a tree is a cut set' — Justify.

(i) Define the order of convergence of an iterative process. What is the order of convergence of the Secaut Method?

(j) Give the formal definition of a Finite State Machine (FSM).

(k) Let G be the grammar with vocabulary V = {S, 0, 1), set of terminals T = {0, 1}, starting symbol S, and productions P = {S $\rightarrow$ 11S, S $\rightarrow$ 0}. What is L(G), the language of this grammar?

(l) State the Arden's theorem on Regular Expression.

(m) What are the advantages of using R–K method for numerical solutions of differential equations over Taylor's Series method?

(n) Use Lagrange polynomial to find the value of f(x) at x = 4 for the table given below.

x	1.5	3	6
f(x)	–0.25	2	20

(o) Find the minimum number of students to be present in a class such that at least nine students are there who were born in the same month.

Group – A

(Discrete Mathematical Structures)

2. (a) Define the term metric in a graph. Prove that the distance between vertices of a convected graph is a metric.

(b) Prove that every circuit has an even number of edges in common with any cut-set

(c) Describe an algorithm with a non-trivial example, for finding a minimum spanning tree from a given weighted undirected graph G. (2+4)+4+6

3. (a) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits.

(b) Define and illustrate with suitable examples the terms eccentricity and centre of a graph G.

(c) Prove that if A and B are independent events, than A^c and B^c are also independent events. 6+(2+2)+6

4. (a) State and prove the generalized Pigeonhole Principle.

(b) There are six processes to be assigned to three processors. In how many ways can you distribute these processors over processors such that no processors will be idle? A processor can accept multiple processes but a single process is to be allocated by a single processor only.

(c) What is an Adjacency Matrix of a graph G? State two of its characteristics. What is a Path Matrix? (2+4)+4+(2+2+2)

5. (a) Define 0(g(n)) and θ(g(n)) notations and give their geometrical and physical interpretations.

(b) Solve the recurrence relation

$a_n = a_{n-1} + 2a_{n-2}$

with $a_0 = 2$ and $a_1 = 7$

(c) Prove that if E_1 and E_2 are events in the sample space S, then :

$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$

(d) Determine whether or not the given pair of well formed propositions are logically equivalent :

(i) $\sim ((A \wedge B) \vee C)$ and $\sim A$

(ii) $((A \rightarrow B) \rightarrow C) \rightarrow \sim(A \vee B)$ and $\sim(\sim(A \wedge C))$ (2+2+2)+4+3+3

Group – B

(Numerical Methods and Algorithms)

6. (a) If h is very small, prove that $\Delta^{n+1}f(x_0) \approx h^{n+1}f^{n+1}(x_0)$

(b) Write an algorithm to find the linear regression for least square fit equation of y over x.

(c) Evaluate $\int_0^1 \cos x\, dx$, taking five intervals. 4+6+6

7. (a) State and prove Lagrange's Interpolation formula for equal intervals.

(b) Find the positive roots of the equation $x^3-3x+1\cdot06 = 0$, by any method, correct to three decimal places.

(c) Write an algorithm for Gauss-Seidel iterative method to solve a system of linear equations.

(2+4)+6+4

Group – C

(Formal Languages and Automata Theory)

8. (a) Design a "mod 2 counter" i.e., an FSM whose input sequence is a sequence (x_1, x_2, ... x_n) of 0_s and 1_s, and whose output sequence (y_1, y_2, ...y_n) indicates the arrival of every other input 1, i.e,

$$y_n \begin{cases} 1 \text{ if } x_n = 1 \text{ and if there have been an even number of input 1s so far.} \\ 0 \text{ in all other cases.} \end{cases}$$

(b) Write a brief note on Chomsky classification of grammar.

(c) Find a grammar generating $\{a^j b^n c^n \,|n \geq 1,\ j \geq 0\}$ 4+6+6

9. Consider the following grammar :

(a) (S is the starting symbol, and other symbols have their usual meaning)

$S \rightarrow AB;\ A \rightarrow A\cdot 1|0;\ B \rightarrow 2B|3$. Identify the grammar according to Chomsky classification of grammar.

(b) State the Pumping Lemma for context free languages. What is its significance?

(c) Consider a Mealy machine given below. Construct a Moore machine equivalent to this Mealy machine. 6+(4+2)+4