2018

MATHEMATICS—HONOURS

Third Paper

(Module – V)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as applicable.

Group - A

(Modern Algebra - II)

Marks - 15

Answer ***any three*** questions:

1. (a) Define an alternating group of degree 'n' and find the order of that group.

(b) Prove that every group of prime order is cyclic. (1+1)+3

2. If $\alpha = (134)(56)(2789)$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 9 & 6 & 4 & 5 & 2 & 3 & 1 \end{pmatrix}$ be two permutations, find $\mu = \beta^{-1}\alpha\beta$. Is μ an even permutation? Can it be expressed as the product of transpositions uniquely? Justify your answer. 5

3. Considering the set of all matrices $M = \left\{ \begin{pmatrix} a & b \\ 2b & a \end{pmatrix}, a, b \text{ rational} \right\}$ as a ring under matrix addition and matrix multiplication, show that M forms a field.

Is M a field when a, b are real numbers? Justify your answer. 3+2

4. Let R be a commutative Ring with unity and without divisors of zero. Prove that if R is finite then R is a field. 5

5. (a) Justify whether the ring Z_6 of all integers modulo 6 is an Integral Domain.

(b) Let F be a field. If $a, b \in F$ such that $b \neq 0$ and $(ab)^2 = ab^2 + bab - b^2$, then prove that $a = 1$.

3+2

Group - B

(Linear Programming and Game Theory)

Marks - 35

Answer ***any five*** questions:

6. (a) Prove that if there is a feasible solution to $Ax = b, r(A) = m$, then there exists a basic feasible solution. State the geometrical equivalence of the above statement.

(b) Given that (1, 3, 2) is a feasible solution of the following equations:

$$2x_1 + 4x_2 - 2x_3 = 10$$

$$10x_1 + 3x_2 + 7x_3 = 33$$

Reduce the above feasible solution to a non-degenerate basic feasible solution. 3+4

7. (a) Prove that the interior I of a convex set S of n-vectors is a convex set.

(b) Three different types of cars A, B and C have been used to transport 60 tons solid and 35 tons liquid substances. A type car can carry 7 tons solid and 3 tons liquid. B type car can carry 6 tons solid and 2 tons liquid and C type car can carry 3 tons solid and 4 tons liquid. The cost of transport are Rs. 500, Rs. 400 and Rs. 450 per car of A, B and C types respectively. Formulate the problem mathematically to obtain the minimum transportation cost. 3+4

8. Use Charne's Big M method to solve the LPP:

Maximize $Z = x_1 + 5x_2$ subject to $3x_1 + 4x_2 \leq 6, x_1 + 3x_2 \geq 3$ where $x_1, x_2 \geq 0$. Also verify the accuracy of the solution by using the graphical method. 4+3

9. (a) Solve the following LPP:

Maximize $Z = 2x_1 + 5x_2$

Subject to $2x_1 + x_2 \geq 12$

$x_1 + x_2 \leq 4$

$x_1 \geq 0$ and x_2 is unrestricted in sign.

(b) Write down the mathematical formulation of an Assignment problem. 5+2

10. (a) If X^* and W^* be any two feasible solutions of the primal, maximize $Z = CX$, subject to $AX \leq b, X \geq 0$ and the corresponding dual, minimize $Z_w = b^T W$, subject to $A^T W \geq C^T, W \geq 0$, respectively and $CX^* = b^T W^*$, then prove that X^* and W^* are the optimal feasible solutions of the primal and dual problems respectively.

(b) State the fundamental theorem of Duality. 5+2

11. Find the optimal assignment profit from the following profit matrix: 7

	D_1	D_2	D_3	D_4	D_5
O_1	2	4	3	5	4
O_2	7	4	6	8	4
O_3	2	9	8	10	4
O_4	8	6	12	7	4
O_5	2	8	5	8	8

12. Find the optimal solution of the following transportation problem and find the minimum cost of transportation. [Use Northwest corner method to obtain the initial basic feasible solution] 7

	D_1	D_2	D_3	
O_1	0	2	1	5
O_2	2	1	5	10
O_3	2	4	3	5
	5	5	10	

13. Convert the following game problem into a Linear Programming Problem:

$$A \overset{B}{\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}}$$

and hence solve it. 3+4

14. Solve graphically the 2×4 game problem whose pay-off matrix is given by

$$\begin{bmatrix} 2 & 2 & 3 & -1 \\ 4 & 3 & 2 & 6 \end{bmatrix}.$$ 7