2018

PHYSICS – HONOURS

Fourth Paper

(Group-A)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer *question no.* **1** and *any* ***four*** from the rest.

Symbols have their usual meanings everywhere.

Given $h = 6{\cdot}626 \times 10^{-34}$ J-sec

$m_e = 9{\cdot}1 \times 10^{-31}$ kg

$C = 3 \times 10^{8}$ m/s

$G = 6{\cdot}67 \times 10^{-11}$ N$-$m^2/kg^2

$K_B = 1{\cdot}38 \times 10^{-23}$J/K

1. Answer ***any five*** of the following.

(a) $\Psi_1(x)$ and $\Psi_2(x)$ are Eigenstates of the Hamiltonian with eigenvalues E_1 and E_2. Is $\Psi(x,t) = c_1\psi_1(x)e^{-iE_1t/\hbar} + c_2\psi_2(x)e^{-iE_2t/\hbar}$ a stationary state?

(b) What is the physical significance of the fact that an operator commutes with the Hamiltonian operator?

(c) The wavefunction of a particle bound inside a one dimensional box is given by $\psi(x) = A\cos Kx + B\sin Kx$. If the two ends of the box are at $x = -a$ and $x = +a$ respectively, then write down the appropriate boundary conditions and modify the wavefunction accordingly.

(d) What do you mean by a state function in thermodynamics? Cite two examples of state function.

(e) Prove that

$$\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V,$$

where the symbols have their usual meanings.

(f) Why is the slope of the fusion curve in the $P - T$ phase diagram of water negative? 2×5

2. (a) Calculate the value of Compton wavelength of an electron.

(b) A single sinusoidal wave can't represent a localized particle— Explain.

(c) The photoelectric work-function W for Li is 2·3 eV. Find the threshold wavelength λ_0 for photoelectric effect. If ultraviolet light of $\lambda = 2000$Å is incident on Li surface, obtain the maximum kinetic energy of photoelectrons and the value of stopping potential.

(d) Draw the polar graph of data obtained from Davisson-Germer experiment indicating clearly the coordinates used. Mention also the potential and angle at which maximum intensity is obtained. What information can we get from this experiment? 2+2+3+3

3. (a) What do you mean by square integrable wavefunction? Cite one example of a wavefunction, which is not square integrable.

(b) A particle is represented by

$$\psi(x) = A(a^2 - x^2) \ , \quad -a \leq x \leq a$$
$$= 0, \text{ otherwise}$$

where A is the normalization constant. Determine the uncertainty in the position of the particle in terms of A and a.

(c) A system has two possible energy values E_0 and $2E_0$ and at a certain instant, the system is in a state in which the expectation value of energy is $\frac{3}{2}E_0$. Calculate the wavefunction in this state, given that ψ_0 and ψ_1 are the wavefunctions corresponding to the two possible energy values E_0 and $2E_0$ respectively. What is the wavefunction after a time t has elapsed? Assume that the eigenvalues E_0 and $2E_0$ are non-degenerate and the relative phase between the states Ψ_0 and Ψ_1 is θ. 2+4+4

4. (a) For a free particle, show that each energy eigenvalue is doubly degenerate.

(b) Show that the momentum operator is a Hermitian operator.

(c) Show that if H is Hermitian then e^{iH} is unitary.

(d) Show that any operator A which has no explicit time dependence follows

$$\frac{d}{dt}\langle A\rangle = \frac{i}{\hbar}\langle [H, A]\rangle,$$

where H is the Hamiltonian operator. 2+3+2+3

5. (a) For a thermodynamic system, show that

$$C_P - C_V = \alpha^2 B_T T V$$

where α is the coefficient of volume expansion at constant pressure and B_T is the isothermal bulk modulus.

(b) State Zeroth law of thermodynamics and briefly explain (qualitatively) the concept of temperature from it.

(c) If E_S and E_T are the adiabatic and isothermal elasticity constants of a gas and γ is the ratio of two specific heats C_P and C_V, then prove that $\gamma = \frac{E_S}{E_T}$. 3+3+4

6. (a) What is free expansion? Show that Joule's coefficient for a gas is given by

$$\mu = \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right].$$

Using this expression, prove that the ideal gas shows no change in temperature in free expansion.

(b) Starting from Gibb's free energy (G), show that

$$H = -T^2\left[\frac{\partial}{\partial T}\left(\frac{G}{T}\right)\right]_P,$$

where the symbols have their usual meanings.

(c) Prove that for the specific heats C_P and C_V

(i) $\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V$

(ii) $\left(\frac{\partial C_P}{\partial P}\right)_T = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_P$ 5+2+3

7. (a) Using Legendre transformation, obtain Gibb's free energy $G(T, p)$ from internal energy $U(S, V)$, where p, V, T, S are the usual thermodynamic parameters.

(b) Prove the following relations where symbols have their usual meanings:

(i) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$

(ii) $\frac{\left(\frac{\partial P}{\partial T}\right)_S}{\left(\frac{\partial P}{\partial T}\right)_V} = \frac{\gamma}{\gamma-1}$

(c) For a thermodynamic system, show that

$$TdS = C_V dT + \frac{\beta T}{K_T}\,dV,$$

where β and K_T are volume expansivity at constant temperature and isothermal compressibility, respectively. 3+4+3

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