2018

PHYSICS — HONOURS

Seventh Paper

(Group – A)

Full Marks – 50

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

Answer ***Question No.1*** and ***any four*** from the rest

1. Answer ***any five*** questions : 2×5

(a) What do a microstate and a macrostate represent in a phase space of a statistical system describing N particles moving in three dimensions?

(b) Show that the average energy of a system in contact with a heat bath can be expressed as

$$\langle E \rangle = -\frac{\partial \ln Z}{\partial \beta}$$ where Z is the partition function and $\beta = \frac{1}{kT}$

(c) Assuming the entropy S is a function of the number of microstates Ω, show that the additive character of S and the multiplicative character of Ω necessarily require $S \propto \ln \Omega$.

(d) Why is displacement current necessary for the consistency in Maxwell's equations?

(e) How can you generate an electric field that is non-conservative in nature?

(f) A 1KW source is emitting waves isotropically. What is the amplitude of the electric field at a distance of 1 km?

2. For a system of N one-dimensional harmonic oscillators, obtain the canonical partition function. Calculate average energy and obtain the quantum limit.

Calculate the canonical partition function for an ensemble of localized magnetic dipoles in an magnetic field. Hence, find out the average dipole moment along the magnetic field. Sketch the function as a function of temperature.

2+3+2+2+1

3. A system of N identical spinless bosons of mass m is in a cubic box of volume $V=L^3$ at temperature $T>0$.

(a) Write down the general expression for the number of particles, n(E), having an energy between ϵ to $\epsilon + d\epsilon$ in terms of their mass, the energy, the temperature, the chemical potential, the volume.

(b) Show that in the limit that the average distance d between the particles is very large compared to their de Broglie wavelength (i.e. $d >> \lambda$) the distribution becomes equal to the classical Boltzmann distribution.

(c) Show the cases where two identical bosons can be distributed among three states. Obtain the Planck's blackbody law from Bose-Einstein statistics.

2+3+(2+3)

4. (a) Derive the density of states as a function of energy ϵ for a free electron gas in one-dimension [linear extent L]. Then, calculate the Fermi Energy ϵ_F at zero temperature for an N electron system.

(b) Calculate the average energy per particle ϵ, for a Fermi Gas at T=0, given that ϵ_F is the Fermi energy for both the non-relativistic (p<<mc) and relativistic (p>>mc) case. (2+3)+(3+2)

5. Consider a linearly polarized light incident from a region of refractive index n_1 onto a material of refractive index $n_2 < n_1$. If the primed and unprimed quantities refer to the reflected and incident amplitudes respectively, then show that –

(a) $\frac{E_1'}{E_1} = \frac{(n_2\cos\theta_1 - in_1\beta)^2}{n_2^2\cos\theta_1 + n_1^2\beta^2}$ where $\cos\theta_2 = i\beta$

(b) Hence, find $\tan\frac{\phi_p}{2} = -\frac{n_1\beta}{n_2\cos\theta_1}$

$$= \frac{\sqrt{\sin^2\theta_1 - \frac{n_2^2}{n_1^2}}}{\left(\frac{n_2^2}{n_1^2}\right)\cos\theta_1}$$

for the p-polarized wave i.e. component of the wave polarized in the plane of incidence. [You can assume the formulas for reflection and transmission coefficients without having to derive them]. 5+5

6. (a) Starting from the curl of the $\vec{E}$ in Maxwell's equation, show that in of $\vec{E}$, the new vector $\vec{E}' = \vec{E} + \frac{\partial\vec{A}}{\partial t}$ can be expressed as a gradient of scalar potential, in case of time dependent fields in vacuum. Write down the modified Poisson equation in terms of V and $\vec{A}$. Obtain the analogous equation for the vector potential $\vec{A}$ from the Ampere's law. [all fields are in vacuum]. 2+2+2

(b) An electron with speed v executes cyclotron motion between the parallel faces of an electromagnet whose field is increased at a rate $\frac{\vec{dB_z}}{dt}$. Determine the tangential acceleration of the electron. 4

7. Obtain an expression for skin depth. Show how it changes with the frequency of the incident wave.

Find out the momentum density stored in an electromagnetic field.

A rectangular wave guide has dimensions 2.28cm×1.01 cm. If the driving frequency is 1.70×10^{10}Hz, find out the TE modes that would propagate through this guide. 4+1+2+3