K(II)-Statistics-H-3A

2018

STATISTICS – HONOURS

Third Paper

Group - A

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Section – I

Answer ***any two*** from ***question nos.*** **1-4** and ***any one*** from ***question nos.*** **5** and **6**

1. Show that $\frac{dF(x)}{dx} = \frac{1}{h}\left[\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \ldots\right]F(x)$, where h is the difference interval. 5

2. Show that for any α, $\Delta \operatorname{Tan}(ax) = \frac{\sin(a)}{\cos(ax)\cos(a(x+1))}$. 5

3. Given

x	0·5	0·6	0·7	0·8	0·9	1·0	1·1
y	0·48	0·56	0·65	0·73	0·80	0·87	0·93

Find $\int_{0.5}^{1.1} y^2 dx$. 5

4. Consider the vector $v = (2\ 3\ 1)'$. Find the vector $u = (u_1, u_2, u_3)'$ which maximizes the dot product with v, subject to the condition that u is of unit length. 5

5. (a) Derive Lagrange's interpolation formula.

(b) Show that the Newton's Forward formula can be obtained as a special case of (a).

(c) Describe the method of iteration for solving an equation in one unknown and discuss its convergence. 6+4+5

6. (a) Show that $f(x, y) = y^2 + x^2y + x^4$ has a minimum at the origin.

(b) Evaluate $\iint \frac{dxdy}{a^2 + x^2 + y^2}$ taken over the region $x^2 + y^2 \leq 1$.

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(c) Consider the transformation $(X_1, X_2) \to (U_1, U_2)$ given by

$$U_1 = \sqrt{-2\, In\, X_1}\, \text{Cos} 2\pi X_2$$

$$U_2 = \sqrt{-2 In X_1}\, \text{Sin} 2\pi X_2$$

Find the Jacobian of the transformation. 5+5+5

Section – II

Answer ***any two*** from ***question nos.*** **7-10** and ***any one*** from ***question nos.*** **11** and **12**.

7. Let the p.g.f. of X be gx(t). Find the p.g.f.s of X+1 and 2X in terms of gx(t). 5

8. Suppose F () is the cdf of the standard logistic distribution. Find an expression for the quantile function F^{-1} (p). 5

9. Suppose events occur in time according to a Poisson process with parameter λ and let $X \sim$ Poisson (λ). Let T denote the length of time until k occurrences. Find the p.d.f. and c.d.f. of T. 5

10. What are equivalent scores. Discuss how it can be obtained from ogives. 5

11. (a) A company insures homes in 3 cities J, K and L. The losses occurring in these cities are independent. The m.g.f. for the loss distributions of the cities are respectively

$M_J(t) = (1-2t)^{-3}$, $M_K(t) = (1-2t)^{-2\cdot5}$ and $M_L(t) = (1-2t)^{-4\cdot5}$.

Let X represent the combined loss from the 3 cities. Calculate $E(X^3)$.

(b) Find the mean and variance of a hypergeometric distribution with parameters N, n and p (symbols having their usual meaning). Write down the limiting form of the distribution when $N \to \infty$.

(c) Let $X \sim N(0, 1)$ independently of $W \sim$ Bernoulli (0·5). Define Y = X if W = 0 and = –X if W = 1. Find the p.d.f. of Y and hence that of X+Y. 5+5+5

12. (a) Suppose $X \sim N(\mu, \sigma^2)$, but is truncated to lie in the interval (a, b), $-\infty < a < b < \infty$. Find the p.d.f. of X and hence find E(X).

(b) Suppose that X has the basic Pareto distribution with shape parameter a. Show that 1/X has the beta distribution.

(c) The random variables X, Y follow a bivariate distribution with joint density

$f(x, y) = 3e^{-(x+y)}$ for $0 < 2x < y < \infty$ and zero elsewhere. Find the density of Z = Y/X

6+4+5