

2018

STATISTICS-HONOURS

Third Paper

Group - B

Full Marks : 50

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

Section - I

Answer *any two* from *question nos. 1-4* and *any one* from *question nos. 5 and 6*

1. Distinguish clearly between *theoretical distributions* and *smapling distributions* citing examples of each. 5
2. If X_1, X_2 and X_3 are i.i.d. $N(0, 1)$, find the distribution of $[(X_1 + X_2)^2 + 2 X_3^2] / [X_1 - X_2]^2$.
(proofs of intermediate results are not necessary) 5
3. If X_1 and X_2 are independent and follow exponential distributions with same mean λ , find the distribution of $X_1 / (X_1 + X_2)$. 5
4. If X_1, \dots, X_n is a sample from a population defined by the p.d.f. $f(x)$, find the joint distribution of $\min \{X_i\}$ and $\max \{X_i\}$. 5
5. For a sample of size n from a $N(\mu, \sigma^2)$ population, let X and s^2 be the sample mean and sample variance (with divisor $n-1$) respectively.
 - (a) Find the joint distribution $f(X, s^2)$.
 - (b) Show that this equals the product of $g(X)$ and $h(s^2)$, where $g(\cdot)$ and $h(\cdot)$ are respectively the marginal distributions of X and s^2 .
 - (c) Hence find the form of the distribution $f(u)$, where $u = n^{1/2} (X - \mu) / s$.
 - (d) What will be the distribution of u^2 ? 5+2+6+2
6. (a) Given a sample of size n from $N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, find the distribution of $Q = (1 - \rho^2)^{-1} [s_1^2 / \sigma_1^2 - 2 s_{12} / \sigma_1 \sigma_2 + s_2^2 / \sigma_2^2]$ where s_1^2 and s_2^2 are the two sample variances and s_{12} is the sample covariance.
- (b) If the errors of a simple linear regression model are i.i.d. $N(0, \sigma^2)$, derive the distributions of the intercept, the slope and the residual variance as estimated from a sample of size n . 6+9

Please Turn Over

Section – II

Answer *any two* from *question nos. 7-10* and *any one* from *question nos. 11 and 12*.

7. Let T_1 and T_2 be two unbiased estimators of θ having the same variance. Find a lower bound to their correlation coefficient in terms of their efficiency. 5
8. Using a sample of size n , find the method of moment estimators of α and p of a Gamma $G(\alpha, p)$ distribution. 5
9. In a library there are N (known) books of which M (unknown) are in English and the rest $(N - m)$ in Vernacular. To estimate M , n books are chosen at random without replacement of which x turns out to be in English. Find the maximum likelihood estimator of M . 5
10. In reporting an interval estimate of the mean μ of a normal distribution, a statistician quotes
- $$P[55.5 \leq \mu \leq 82.5] = 0.95.$$
- Is the statement correct? If not, how will you re-state it? –Discuss. 5
11. (a) In estimating the parameter θ , Statistician A suggests using the minimum variance unbiased estimator T_A while Statistician B suggests using the minimum mean-square-error estimator T_B . Which of the two will you prefer and why?
- (b) State Cramer-Rao Inequality and discuss its implications.
- (c) What implication does the Rao-Blackwell Theorem have in the context of minimum variance unbiased estimators?
- (d) Let $f(x) = e^{-(x-\theta)}$ if $\theta < x < \infty$ and $= 0$ otherwise.
- If the Cramer-Rao inequality is applicable, find a lower bound of the variance of an unbiased estimator of θ . 3+4+3+5
12. Let X_1, \dots, X_n be a random sample from a Rectangular $(0, \theta)$ distribution and let T_1 be the sample mean.
- (a) Find the maximum likelihood estimator (T_2) of θ .
- (b) Are T_1 and T_2 unbiased for θ ? If not, find unbiased estimators of θ based on each.
- (c) Which of the two unbiased estimators will you prefer and why?
- (d) Find the maximum likelihood estimator of θ if X_1, \dots, X_n is a random sample from a Rectangular $(-\theta, \theta)$ distribution. 3+4+5+3
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