2018

STATISTICS — HONOURS

Sixth Paper

(Group – A)

Full Marks – 50

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

Answer ***any four*** questions from ***Question No. 1*** to ***Question No. 8***, and answer ***any two*** questions from ***Question No. 9*** to ***Question No. 12***

1. For a population containing 5 units, denoted by U = (1, 2, 3, 4, 5), write all the simple random samples without replacement of size 3 (SRSWOR(5, 3)). Find the probability of drawing (1, 2, 3) as a sample. 5

2. Let π be the proportion of an attribute in a population size N. For an SRSWR(N, n), find an unbiased estimator of π with its variance. 5

3. For an SRSWOR(N, n), show that the sample mean is the best linear unbiased estimator of the population mean. 5

4. Describe the basic principles of sample surveys. 5

5. In connection with design of experiments, describe the terms (a) experiment (b) treatment (c) experimental error. 5

6. Describe uniformity trial with its use. 5

7. Describe a situation where a latin square design (LSD) is appropriate. Give a brief analysis of an LSD of order s×s. 5

8. In a 2^2 – factorial experiment, describe how the simple and factorial effects are defined. 5

9. (a) For an SRSWOR(N, n), find the probability that units u_i and u_j occur together in the samples.

(b) Let t_i be the number of times that an unit u_i occurs in an SRSWR(N, n). Show that $(t_1, t_2, \ldots, t_N)$ follows a multinomial distribution with parameters to be obtained.

Use the above result to show that the sample mean is an unbiased estimator of the population mean. Also from the above results, find the variance of the sample mean. 7+8

10. (a) In an ANCOVA conducted in an RBD with r replications and t treatments, show that the average variance of estimators of all elementary treatment differences is given by

$$\overline{V} = \frac{2\sigma^2}{r}\left[1 + \frac{T_{xx}}{(t-1)E_{xx}}\right],$$

where T_{xx} is the treatment ss for x observations and E_{xx} is the error ss for the x observations.

(b) Describe a situation where a split plot design (SPD) is suitable. Give analysis of an SPD where the levels of the whole plot treatment are arranged according to RBD. 7+8

11. (a) Prove the following matrix result :

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$

Starting from the vector of treatment effects,

show that Yates' method of computation of factorial effects in terms of them in a 2^2-factorial set-up follows from the above relation.

(b) For a $(2^4, 2^2)$ confounded design, give a plan in appropriate number of replicates where all the 2–, 3 – and 4 – factor effects are balanced. Give the control block for any one of the replicate. 7+8

12. (a) Show that in SRSWOR(N, n), an unbiased estimator of $V(\overline{y})$ is given by $\hat{V}(\overline{y}) = \frac{N-n}{Nn}s^2$, where s^2 is given by $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2$.

(b) Mentioning the situations where strip plot design is suitable, give analysis of such design. 7+8