2018

STATISTICS — HONOURS

Sixth Paper

(Group – B)

Full Marks – 50

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

Unit – I

(Time Series Analysis)

(Marks – 25)

Answer ***any two*** questions from ***Question Nos.*** **1** to **4,** and ***any one*** from ***Question Nos.*** **5** and **6**

1. Describe the purpose of analysing time-series data. 5

2. How can you use the method of least squares to fit an exponential equation to time-series data? Is this method applicable to fit modified exponential trend? 5

3. No appreciable change has been observed in the total annual rainfall over the years in Kolkata. Using this information, discuss how you will find the seasonal variation in rainfall in Kolkata on the basis of monthly data observed over eight years. 5

4. Describe the method of exponential smoothing as applicable in the analysis of time-series data. 5

5. (a) Does the trend-component has any role in determining seasonal variation of a time-series? Specify appropriate models used in the analysis of time-series data in your answer.

(b) Show that trend-value obtained by applying moving-average method is actually a use of fitting a polynomial equation to time-series data. Use equations involving degree one and two of the time component. 7+(3+5)

6. (a) Estimate the parameters of an AR(2) process using the Yule-Walker equations.

(b) What do you mean by changing seasonal pattern? Discuss different types of changing seasonal patterns.

(c) What do you mean by a time-series to be weak stationary? 7+5+3

Unit – II

(Sample Survey)

(Marks – 25)

Answer ***any two*** questions from ***Question Nos.*** **7** to **10,** and ***any one*** from ***Question Nos.*** **11** and **12**

7. Make a comparison between a complete enumeration and a sample survey. Also state situations where complete enumeration must be implemented. 5

8. In case SRSWOR of size n_h is adopted for the h^{th} stratum of size N_h, show that an unbiased estimator of the population proportion P is given by

$$\hat{P} = \sum_{h=1}^{L} W_h p_h$$

and that $$V(\hat{P}) = \sum_{h=1}^{L} W_h^2 \frac{1-f_h}{n_h} \frac{N_h P_h \left(1-P_h\right)}{N_h - 1},$$

where $W_h = \frac{N_h}{N}$, $f_h = \frac{n_h}{N_h}$, and p_h and P_h are, respectively, the sample proportion and the population proportion in the h^{th} stratum. 2+3

9. In case of a two-stage sampling scheme with equal-sized first-stage units, suggest an unbiased estimator of the population mean and derive its sampling variance. [Specify an appropriate sampling procedure you may adopt in this case.] 5

10. In case of estimating the population mean of a study variable y, obtain an estimator of the gain (defined by sampling variance of estimator minus that of the other) due to stratification over simple random sampling on the basis of random data on a study variable y. 5

11. (a) In a population of size N, the variate value of one of the units is known to be y_1. An SRSWOR of size n is selected from the remaining $(N-1)$ units and the sample mean $\bar{y}'_n$ is used in the estimator t_1, given by $t_1 = y_1 + (N-1)\bar{y}'_n$. Show that t_1 is unbiased for the population total and has a smaller variance than that of the unbiased estimator $t = N\ \bar{y}_n$ based on an SRSWOR of size n taken from the entire population.

(b) Describe how you would use the information on an auxiliary variable x correlated with the study variable y to improve the simple unbiased estimate of the total of y-values in simple random sampling without replacement. (2+2+6)+5

12. (a) For systematic sampling, obtain an estimator of the population mean. Show that the variance of the estimator can be expressed as

$$\frac{\sigma^2}{n}\left[1+(n-1)\rho_c\right],$$

where σ^2 is the population variance and ρ_c is the correlation coefficient between pairs of sample units in the same sample of size n.

(b) Show that for a hypothetical population where the values of the population units have a linear trend, systematic sampling is more efficient than simple random sampling. 8+7