2018

STATISTICS – HONOURS

Fifth Paper

(Group – A)

Full Marks – 50

The figures in the margin indicate full marks
Candidates are required to give their answers in their own words as far as practicable

Notations have their usual significance

Unit – I

1. Answer ***any two*** questions : 5×2

(a) Define multivariate data. Distinguish between sample multiple correlation coefficient and population multiple correlation coefficient.

(b) Suppose $r_{0i} = \alpha$ for $i = 1, 2, ..., p$ and $r_{ij} = \beta$ for $i, j = 1, 2, ..., p$, $i \neq j$. Find the expression for $r_{0.123...p}$ in terms of α and β. If $\alpha = 0$, find the value of $r_{0.123...p}$ and comment.

(c) Suppose $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right)$, Σ is positive definite. Find the moment generating function of $\underset{\sim}{X}$.

(d) Give intuitive justification of

$$0 \leq \rho_{1.23...\overline{p-1}} \leq \rho_{1.23...p} \leq 1.$$

Discuss all the equality cases.

2. Answer ***any one*** question :

(a) (i) Define sample partial correlation coefficient. Discuss its importance. Show that

$$\left(1 - r_{1.23...p}^2\right) \leq \left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)...\left(1 - r_{1\overline{p-1}.23...\overline{p-2}}^2\right)$$

Discuss the equality case. 9

(ii) Explain the concept of ellipsoid of concentration. 6

(b) (i) Suppose $\underset{\sim}{X}^{p\times 1}$ is a random vector with mean vector $\underset{\sim}{\mu}$ and dispersion matrix Σ. Find $E\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)' \Sigma^{-1}\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)$ and verify which of the probabilities $P\left[\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)' \Sigma^{-1}\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right) > 3p\right]$ and $P\left[\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)' \Sigma^{-1}\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right) < 3p\right]$ is larger. 2+3

(ii) In (i) if $p=3$, $\underset{\sim}{\mu}=(0,0,0)'$ and $\Sigma=\begin{pmatrix}1 & c & 0\\ c & 1 & c\\ 0 & c & 1\end{pmatrix}$, then is it possible to find c for which $\underset{\sim}{a}'\underset{\sim}{X}$ and $\underset{\sim}{b}'\underset{\sim}{X}$ are independently distributed, where $\underset{\sim}{a}'=(1,-1,-1)$ and $\underset{\sim}{b}'=(1,1,1)$? Justify your answer. 4

(iii) Define probability density function of a random vector. Give an example of a probability density function of a 5-variate random vector (with justification) which is non-normal but all its marginals are normal. 6

Unit – II

3. Answer ***any two*** questions : 5×2

(a) Let $X_1, X_2, ..., X_n$ be independent and identically distributed uniform $(0,\theta)$ random variables, $\theta>0$. Find two "consistent and unbiased" estimators for θ. Which one would you prefer and why?

(b) Using the definition of consistency, show that if T_n is consistent for θ, then e^{T_n} is consistent for e^{θ}.

(c) Give an example of a sequence of random variables $\{X_n\}$ which converges in distribution to Z, a standard normal variable and also give an example of a sequence of random variables $\{Y_n\}$ which does not converge in distribution to Z. (Give reasons).

(d) Explain the concept of delta method with an example.

4. Answer ***any one*** question :

(a) (i) Find the large sample distribution of Pearsonian chi-square for multinomial proportion. 9

(ii) Describe how you can use Pearsonian chi-square statistic to test whether a large sample of observations is a random sample from Normal (1,1) distribution. 6

(b) (i) Suppose (X_i, Y_i), $i=1,2,...,n$ is a random sample from $N_2(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho)$. Find a variance stabilizing transformation for the sample correlation r. Give a real life application of it mentioning the advantage of this transformation. 6

(ii) State central limit theorem for independent and identically distributed random variables. Discuss its importance in statistical inference. 4

(iii) Write a short note on large sample standard error of sample coefficient of variation. 5