2018

STATISTICS – HONOURS

Fifth Paper

(Group – B)

Full Marks – 50

The figures in the margin indicate full marks
Candidates are required to give their answers in their own words as far as practicable

Answer ***Question No. 1*** and ***any two*** questions from the rest

1. Answer ***any four*** questions :

(a) Make a comparative study between —

(i) Simple and composite hypotheses

(ii) Type-I and Type-II errors. 5

(b) How do you define p-value of a two-tailed test in general? Hence, get the p-value of a two-tailed test when the null distribution of the test statistic is symmetric about zero. 5

(c) On the basis of road accident data, discuss how to perform an exact test to verify whether the traffic control system has improved on a certain year as compared with its previous year. 5

(d) Suppose the weights on n pigs before and after application of a special food supplement in a farm are recorded. Under suitable assumption(s), provide a test procedure for testing whether the variabilities in the pigs' weights before and after applying food supplement remain unchanged. 5

(e) Describe a testing procedure using ANOVA technique whether the average eye-estimation in measuring some physical characteristic can be expressed as a linear function of their exact measurements. 5

(f) What do you mean by valid error in the context of ANOVA? Explain briefly the role of valid error in hypothesis testing of ANOVA. 5

(g) Suppose a random sample of size n_1 is drawn from common pdf $f(x,\theta_1)=\frac{1}{\theta_1}e^{-x/\theta_1},\ x>0$, and another random sample of size n_2 is drawn from common pdf $f(x,\theta_2)=\frac{1}{\theta_2}e^{-x/\theta_2},\ x>0$. Find a $100(1-\alpha)\%$ confidence interval for θ_2/θ_1. 5

(h) Based on age data of n patients appearing in a clinic, discuss how to construct an interval within which the true value of the population median age can lie with at least 95% confidence. (Note that age distribution of the patients is completely unknown but known to be continuous). 5

2. Suppose $x_1,...,x_n$ are iid with a common Rayleigh pdf given by

$$f(x;\theta)=\frac{2}{\theta}xe^{-x^2/\theta},\ x>0$$

where $\theta(>0)$ is unknown.

(a) Derive an UMP level-α test for testing $H_0:\theta=\theta_0$ vs $H_1:\theta<\theta_0$, where θ_0 is a known positive number.

(b) Do you think that an LR level-α test for testing $H_0':\theta\geq\theta_0$ vs $H_1':\theta<\theta_0$ will coincide with the UMP test obtained under (a)? Justify.

(c) Show that an MP or UMP test is always unbiased. 7+5+3

3. (a) Suppose X be an observable random variable with its pdf $f(x)$, $x\in\mathbb{R}$. Describe an MP level-α test for testing

$$H_0:f(x)=f_0(x)=\frac{1}{\pi(1+x^2)}\text{ vs }H_1:f(x)=f_1(x)=\frac{1}{2}e^{-|x|}$$

Also, perform the power calculation.

(b) Assuming consumption-expenditure to be linearly dependent on income, develop a testing procedure to test whether marginal propensity to consume (MPC) remains same in two consecutive decades. (MPC means change in consumption-expenditure for unit increase in income).

(c) Define contrast of a number of effects. Show that maximum $(n-1)$ orthogonal contrasts are possible from n effects. 5+6+4

4. (a) Based on a random sample of size n drawn from $N(\mu,\sigma^2)$ distribution with known μ, derive the shortest expected length confidence interval for σ^2.

(b) Suppose, with five random sample observations, Wilcoxon signed rank test statistic (W) is used to test $H_0:\theta=\theta_0$ against $H_1:\theta>\theta_0$, where θ is the population median. If H_0 is rejected for $W>12$, show that the test is unbiased.

(c) Also, show that the null distribution of W, mentioned in part (b) is symmetric. 7+5+3

5. (a) When does a test statistic called distribution free? Why is such a test statistic required in non-parametric testing?

(b) What do you mean by an unbiased confidence set?

(c) Suppose $(X_1, X_2, X_3, X_4)' \sim MN(m\ ;\ p_1, p_2, p_3, p_4)$, $\sum_{i=1}^{4} p_i = 1$.

Provide a size-α testing procedure to test

$H_0 : p_i = \frac{1}{4}$ for all $i = 1(1)4$ vs $H_1 : p_1 - p_2 = \frac{p}{2}$, $p_3 = p_4 = \frac{1-p}{2}$.

(d) Suppose in a manufacturing house θ_0 denotes the target quantity content in each packet of product. To check whether packaging is done properly or not two experimenters independently tested two sets of null vs alternative hypotheses :

(i) $H_0 : \theta - \theta_0$ vs $H_1 \cdot \theta < \theta_0$, (ii) $H_0 : \theta = \theta_0$ vs $H_1 : \theta \neq \theta_0$,

and unfortunately reached to contradictory decisions. If you are not biased to any one of them, explain how rationally you can reach to final decision from there.

3+2+6+4