2020

COMPUTER SCIENCE — GENERAL

Paper : DSE-A-2

(Operations Research)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Day 1

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Define a linear programming problem (LPP).

(b) When is a solution to an LPP called a feasible solution?

(c) When is a solution to an LPP called an optimal solution?

(d) What are slack and surplus variables?

(e) When is a solution to a system of simultaneous equations called a degenerate solution?

(f) What do you understand by 2-person zero sum game?

(g) State the primal-dual relationship.

(h) Name two methods for solving the transportation problem.

2. (a) Obtain all the basic solutions to the following system of linear equations :

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5$$

(b) Use Simplex method to solve the following LPP :

$$\text{Max } Z = 7x_1 + 5x_2$$

subject to the conditions,

$$x_1 + 2x_2 \leq 6$$

$$4x_1 + 3x_2 \leq 12$$

$$x_1 , x_2 \geq 0.$$

5+5

3. (a) Obtain the dual problem of the following L.P.P :

$$\text{Max } Z = x_1 - 2x_2 + 3x_3$$

subject to

$$-2x_1 + x_2 + 3x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 1$$
$$x_1,\ x_2,\ x_3 \geq 0.$$

(b) Prove that the dual of the dual of an LPP is its primal. 5+5

4. (a) Prove that a necessary and sufficient condition for the existence of a physical solution to a $m \times n$ Transportation Problem is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

where a_i and b_j denote the availability and requirement at i^{th} origin and j^{th} destination respectively.

(b) Solve the following T.P. to obtain the initial basic feasible solution using Vogel's method.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

5+5

5. (a) Give a mathematical formulation of the Assignment Problem (A.P.).

(b) Solve the following assignment problem

	I	II	III	IV
A	15	14	12	16
B	23	22	25	24
C	31	34	32	33
D	21	32	44	53

where A, B, C, D are 4 jobs assigned to the machines I, II, III, IV.

Find an allocation of jobs to machines so that the total cost of processing is minimum. 5+5

6. (a) Explain the Maxmin principle used in Game Theory.

(b) Solve the game whose pay-off matrix is given by :

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \text{Player A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & -1 \\ 1 & 3 & -2 \end{bmatrix} \end{array}$$

5+5

7. (a) Explain the graphical method for solving an LPP involving two variables.

(b) Solve graphically the following LPP.

Max $Z = 3x_1 + 2x_2$

subject to

$-2x_1 + x_2 \leq 1$

$x_1 \leq 2$

$x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0.$ 5+5

8. (a) Briefly mention the steps to solve a T.P. using North-West Corner rule.

(b) Obtain the initial basic feasible solution using N.W. Corner rule.

	D	E	F	G	Available
A	7	9	3	2	16
B	4	4	3	5	14
C	6	4	5	8	20
Requirement	11	9	22	8	

5+5