2020

PHYSICS — HONOURS

Seventh Paper

(Group - A)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) State the postulate of classical statistical mechanics.

(b) The description of the microcanonical ensemble is given in terms of N, V and the energy between E and $E + \Delta E$, $\Delta E << E$. Why do we need the spread ΔE in energy?

(c) What is thermal wavelength?

(d) Explain the physical significance of the Poynting Vector.

(e) Show that the electric field vector can be written as $\bar{E} = -\bar{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$, in case of a time varying electromagnetic field, where ϕ and $\bar{A}$ are the scalar and vector potentials respectively.

(f) Find an expression for velocity of light in terms of permittivity and permeability starting from Maxwell's equations.

2. (a) State the theorem of equipartition of energy and derive its mathematical expression.

(b) Find the average energy (using the above theorem) of a system of N ideal monatomic molecules. (2+5)+3

3. In how many ways n fermions can be distributed among g non-degenerate levels? Obtain the Fermi-Dirac distribution function at a temperature T using the Boltzmann relation for entropy $S = k \log W$, where the symbols have their usual meanings. Obtain an expression for electronic specific heat for metals at low temperature. 2+4+4

4. Starting from Planck's law for black body radiation, show that the emissive power of a black body is proportional to the fourth power of its absolute temperature. You may use the following results.

$$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$$

A spherical black body of radius 1cm is enclosed in a evacuated chamber. If the chamber is at a temperature 300K, find out the amount of heat that must be supplied per second to the black body to keep it at a temperature 1000K. Neglect conduction of heat. $\left(\sigma = 5.67 \times 10^{-8} W/m^2/K^4\right)$ 5+5

5. In a current free region $B_x = ax + bz$ and $B_y = ax + cy$. Find B_z assuming all currents are outside and $\vec{\nabla} \cdot \vec{j} = 0$. Obtain the reflection coefficient for normal incidence for an electromagnetic wave. Calculate the reflection coefficient for an electromagnetic wave of frequency 10 GHz when it is incident normally on a metal surface of conductivity $6 \times 10^7 (\Omega m)^{-1}$. Take $\epsilon \approx \epsilon_0$ and $\mu \approx \mu_0$. What is Brewster's angle?
2+4+2+2

6. Consider free electrons in a conductor not bound to any particular atom. Under the action of the field $E = E_0 e^{-i\omega t}$ with the charge 'q', mass 'm', 'γ' the damping constant and 'f' the number of free electrons per atom, write down

(a) the equation of motion of the electrons

(b) solution of the equation

(c) an expression for the current J

(d) expression for real part of J

(e) physical significance of J being complex

(f) expression for the conductivity σ. 1+2+2+1+2+2

7. Starting with the Lorentz force law

$$\vec{F} = \frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{V} \times \vec{B}\right)$$

(a) Write down $\vec{F}$ using the vector and scalar potentials.

(b) Show that $\frac{d\vec{A}}{dt} = \frac{\partial \vec{A}}{\partial t} + \left(\vec{V} \cdot \vec{\nabla}\right)\vec{A}$.

(c) Show that $\frac{d}{dt}\vec{p}_{canonical} = -\vec{\nabla}U$, where $\vec{p}_{canonical} = \vec{p} + q\vec{A}$ and $U = q\left(\phi - \vec{V} \cdot \vec{A}\right)$. 2+2+(3+3)

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