2020

PHYSICS — HONOURS

Fifth Paper

Full Marks : 100

The figures in the margin indicate full marks.

Candidates are required to give their answers in thier own words as far as practicable.

1. Answer ***any five*** of the following : 4×5

(a) Define generalized coordinates. A particle is moving on the surface of a sphere. Mention its generalized coordinates.

(b) Show that angular momentum is conserved for the Lagrangian $L = \dot{r}^2 + \vec{r}.\dot{\vec{r}} + r^2$.

(c) Calculate the Hamiltonian for a Lagrangian $L(x, \dot{x}) = \frac{1}{2} x\dot{x}^2 - V(x)$.

(d) Muons at their rest frame have lifetime $2{\cdot}3\times10^{-6}$s while those in cosmic shower have lifetime 16×10^{-6}s measured from earth. Find the speed of the muons in cosmic shower.

(e) If A_i and B_j are arbitrary covariant vectors and $c^{ij}\,A_iB_j$ is an invariant, prove that c^{ij} is a contravariant tensor of rank two.

(f) Does the density of an object change as its speed increases? If yes, by what factor?

(g) Find the eigenvalues and eigenfunctions of the angular momentum operator $\hat{L}_z = i\hbar\frac{\partial}{\partial\phi}$.

(h) Show that for a potential $V(-\vec{r}) = V(\vec{r})$, the eigen functions of the Hamiltonian must be of even or odd parity.

(i) A particle in a one-dimensional harmonic oscillator potential is described by a wave function $\psi(x, t)$. If the wave function changes to $\psi(\lambda x, t)$, then the expectation value of the kinetic energy T will change to what value?

(j) In a Stern-Gerlach experiment, a collimated beam of neutral atoms is split up into seven equally spaced lines. What is the total angular momentum of the atom?

(k) Explain why is pure vibrational spectra observed only in liquids.

(l) Find the magnetic dipole moment of the state $^2D_{3/2}$.

Group - A

Section - I

(Classical Mechanics II)

Answer ***any one*** question.

2. (a) State Kepler's laws of planetary motion.

(b) A particle of mass m moves under the action of a central force $f(r)\hat{r}$. Show that the equation determining the orbit of the particle is

$$\frac{l^2}{mr^2}\left(\frac{d^2(1/r)}{d\theta^2}+\frac{1}{r}\right)=-f(r)$$

where θ is the azimuthal angle and l is a constant of motion. Hence show that for an inverse square force, the trajectory is a conic section. 8+(8+4)

3. (a) State with reasons if the constraints in the following systems is Holonomic or not : (i) Two point masses connected by a massless rigid rod. (ii) The molecules of a gas within a container.

(b) The Lagrangian for a system is given by $L = \frac{1}{2}\dot{q}^2 - q\dot{q} + q^2$. Set up the Hamiltonian for the system and hence find the momentum p conjugate to q.

(c) The length of a plane simple pendulum changes with time such that $l = a + bt$ where a and b are constants. Find the Lagrangian equation of motion. Obtain the Hamiltonian and show that it is not invariant under time-translation i.e. $\frac{\partial H}{\partial t} \neq 0$. 6+6+8

4. (a) Set up Euler's equation of hydrodynamics for an incompressible fluid.

(b) An incompressible fluid of density ρ flows through a horizontal pipe of circular cross-section. The flow is constricted at one place where the radius is reduced from r_1 to r_2. If the difference of pressure before and after the constriction is Δp, show that the rate of flow of the liquid is

$$\sqrt{\frac{2\pi^2 r_2^4 r_1^4 \Delta p}{\rho\left(r_2^4 - r_1^4\right)}}$$

(c) What is a cyclic coordinate? Show that for each cyclic coordinate there exist a constant of motion. 6+8+6

Section - II

(Special Theory of Relativity)

Answer ***any one*** question.

5. (a) Define Lorentz invariant proper time interval $d\tau$.

(b) Find matrices (i) $[g_{ij}]$ and (ii) $[g^{ij}]$ corresponding to $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$ consistent with the definition $ds^2 = g_{ij}dx^idx^j$.

(c) Two lumps of clay each of rest mass m_0 move towards each other with equal speed $\frac{3}{5}c$ and stick together. What is the mass of the composite lump? 4+(4+4)+8

6. (a) Show that it is impossible for an isolated free electron to absorb or emit a photon.

(b) Show that the scalar product $A_\mu B^\mu$ of two four vectors A^μ, B^μ is invariant under Lorentz transformation.

(c) For a particle of rest mass m_0 and momentum p, show that the kinetic energy is given by

$$E = \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2$$ 6+6+8

7. (a) Lagrangian of a one-dimensional, relativistic harmonic oscillator of rest mass m, is

$$L = mc^2\left(1 - \sqrt{1-\beta^2}\right) - \frac{1}{2}kx^2 .$$

Obtain the Lagrange equation of motion and show that it can be integrated to yield

$$E = mc^2 + \frac{1}{2}ka^2$$

where a is the maximum displacement from equilibrium of the oscillating particle.

(b) Calculate the radius of the orbit of an electron of energy E moving at right angles to an uniform magnetic field B.

(c) In a coordinate system with coordinates x^μ, the invariant line element is $ds^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta$. If the coordinates are transformed $x^\mu \to \bar{x}^\mu$. Show that the line element is $ds^2 = g_{\bar{\mu}\bar{\nu}}d\bar{x}^\mu d\bar{x}^\nu$ and express $g_{\bar{\mu}\bar{\nu}}$ in terms of the partial derivatives $\partial x^\mu / \partial \bar{x}^\nu$. For two arbitrary 4– vectors U and V, show that,

$$U.V = U^\alpha V^\beta \eta_{\alpha\beta} = U^{\bar{\alpha}} V^{\bar{\beta}} g_{\bar{\alpha}\bar{\beta}}$$ (4+4)+4+(4+4)

Group - B

Section - I

(Quantum Mechanics II)

Answer ***any one*** question.

8. (a) The wave function of an excited state of a one-dimensional linear harmonic oscillator is of the form

$$\psi(x) = Axe^{-\alpha^2x^2/2}$$

(i) Calculate the expectation value of the momentum operator for this state.

(ii) Sketch the wave function and the corresponding probability density function.

(iii) Write down the energy eigenvalue for this state.

(b) A spin - 0 particle of mass 'm' is in a three dimensional isotropic well described by $V(r)=\frac{1}{2}m\omega^2 r^2$ where $r^2 = x^2 + y^2 + z^2$. How many states will have energy $\frac{7}{2}\hbar\omega$? (8+4+2)+6

9. (a) Let two spin $-\frac{1}{2}$ states of an electron are χ_α, χ_β. What will be the normalized singlet and triplet spin states formed by two such electrons?

(b) Evaluate the commutators (i) $[L_x^2 + L_y^2, L_z^2]$ and (ii) $[L_z, \sin 2\phi]$ (in usual notations).

(c) Find the energy eigenvalues and the normalized wave functions of a particle confined in a one-dimensional box extending over the region $-L/2 < x < L/2$. 6+(4+4)+6

10. (a) Find the expectation value of an operator $\hat{A}$ for a state $\psi = \frac{1}{\sqrt{2}}\phi_1 + \sqrt{\frac{2}{5}}\phi_2 + \frac{1}{\sqrt{10}}\phi_3$ where ϕ_1, ϕ_2 and ϕ_3 are three orthonormal eigen states of $\hat{A}$ such that $\hat{A}\phi_n = n^2\phi_n$.

(b) Consider a potential barrier

$$V(x) = \begin{cases} 0 & \text{for } x<0 \text{ and } x>a \\ V_0 & \text{for } 0<x<a \end{cases}$$

where V_0 is positive. When a particle of mass m and energy $E(0 < E < V_0)$ approaches the barrier from the left, show that the transmission coefficient is

$$\left[1 + \frac{V_0^2 \sin h^2(\beta a)}{4E(V_0 - E)}\right]^{-1}$$

where $\beta = \sqrt{2m(V_0 - E)}/\hbar$. 8+12

Section - II

(Atomic Physics)

Answer ***any one*** question.

11. (a) Deduce an expression for the Lande'g factor of an atom.

(b) Write the spectral symbol of the term with $s = 1/2$, $j = 5/2$, and Lande g factor 6/7.

(c) The quantum numbers of two electrons in a two-valence electron atom are

$$n_1 = 6,\ l_1 = 3,\ s_1 = \frac{1}{2};\ n_2 = 5,\ l_2 = 1,\ s_2 = \frac{1}{2}.$$

(i) Assuming LS coupling, find the possible values of L and hence J.

(ii) Assuming JJ coupling, find possible values of J. 8+6+6

12. (a) The energy levels corresponding to the pure rotational spectrum of diatomic molecules is given by $E_j = BJ(J+1)$, where B is a constant. If λ_1 and λ_2 be the wavelengths of lines corresponding to transitions $J \rightarrow J+1$ and $J+1 \rightarrow J+2$ respectively, find B in terms of these wavelengths.

(b) Assuming that the hydrogen molecule behaves like a harmonic oscillator with a force constant $k = 573$ N/m, find the vibrational quantum number corresponding to its 4.50 eV vibrational energy level, given that the mass of the H atom $= 1.67 \times 10^{-27}$ kg and $h = 6.63 \times 10^{-34}$ Js.

(c) For normal Zeeman effect in hydrogen, explain how Lorentz triplet occurs. 6+8+6

13. (a) What is the role played by an optical resonator in a laser system?

(b) The relative population of two energy states at room temperature $T = 300$K is $1/e$. Determine the wavelength of radiation emitted due to transition between the states.

(c) Consider a 4-level system in which lasing transition occurs between the levels $2 \rightarrow 1$. Show that a necessary condition for population inversion is $\gamma_{10} > \gamma_{21}$ where γ stands for spontaneous decay rate and 0 denotes ground state.

(d) What is Raman effect? What is its practical application? 4+4+6+(4+2)