2020

STATISTICS — HONOURS

Fifth Paper

(Group - A)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Unit - I

1. Attempt ***any one*** question from the following : 10×1

(a) Let $(Y, X_1, X_2, ..., X_p)'$ be any $p+1$ component random vector. Then with the usual notation of multiple correlation coefficient, show that

$$\rho^2_{y.12...p} \geq \rho^2_{y.23...p} .$$

(b) Suppose $R^{p\times p}$ is the correlation matrix of the random vector $X^{p\times 1}$. Show that the determinant of R cannot exceed 1.

(c) Let X and Y be two p-variate random vectors. Find B such that $Z = X + BY$ and Y are uncorreleted, where B is a matrix of appropriate order. Comment on the particular case where X and Y are both normal distribution.

(d) Suppose $(N_1, N_2, ..., N_{2m+1})' \sim$ Multinomial $(n|\pi_1, \pi_2, ..., \pi_{2m+1})$, $0 < \pi_i < 1$, $\forall i = 1, 2, ..., 2m+1$ with $\pi_1 + \pi_2 + ... + \pi_{2m+1} = 1$. Let $Y_1 = \sum_{i=1}^{m} N_i, Y_2 = \sum_{i=m+1}^{2m} N_i$. Determine the joint distribution of Y_1 and Y_2.

2. Attempt ***any one*** question from the following : 15×1

(a) Establish that $\rho^2_{1.23...p} = 1-\left(1-\rho^2_{12}\right)\left(1-\rho^2_{13.2}\right)...\left(1-\rho^2_{1p.23...(p-1)}\right)$.

(b) Let a random vector $X = (X_1, X_2, ..., X_p)'$ have $N_p(\mu, \Sigma)$ distribution. Find the moment generating function of X and also that of $Y = BX$, where B is a $q\times p$ matrix of rank $q(\leq p)$, and identify it as a moment generating function of a multivariate distribution.

(c) If $(X_1, X_2, ..., X_k)'$ has multinomial distribution with parameters $(n, p_1,..., p_k)$, then prove that

$$\rho_{12.34...j} = -\sqrt{p_1 p_2}(1-p_1-p_3-...-p_j)^{-\frac{1}{2}}(1-p_2-p_3-...-p_j)^{-\frac{1}{2}},\ j=3,\ 4,\ ...,\ k$$

(d) Describe the ellipsoid of concentration in p dimensions.

Unit - II

3. Attempt ***any one*** question from the following : 10×1

(a) Explain 'convergence in distribution' of a sequence of random variables. If X_i's are i.i.d. with mean 0 and variance 1, then argue that

$$Y_n = \frac{\sqrt{n}\left(X_1 + ... + X_n\right)}{X_1^2 + ... + X_n^2}$$

is asymptotically $N(0, 1)$.

(b) Derive the large sample standard error of the sample median based on a random sample of size n drawn from an exponential distribution with mean θ.

(c) Find the large sample covariance between sample mean and variance on the basis of a sample drawn from a univariate continuous symmetric distribution and comment.

(d) If r is the sample correlation coefficient of random sample of size n drawn from a bivariate normal distribution with correlation coefficient ρ, derive a suitable variance stabilizing transformation of r. Also find an approximately $100(1-\alpha)\%$ confidence interval for ρ.

4. Attempt ***any one*** question from the following : 15×1

(a) Derive the large sample distribution of Pearsonian Chi-square statistic.

(b) Based on a random sample of size n drawn from Uniform (0, 1) distribution, determine the distribution of sample range and also find the mode of this distribution.

(c) Discuss the method and use of transformation of statistic to make its asymptotic variance independent of unknown parameters.

(d) Suppose S_i^2, $i = 1, 2, ..., K$ are the sample variances of independent samples of sizes, respectively, n_1, n_2, ..., n_K from K univariate normal populations with variances σ_i^2, $i = 1, 2, ..., K$. Derive a large sample test procedure for testing of homoscedasticity of K populations. Also, under the assumption of homoscedasticity, obtain an approximately $100(1-\alpha)\%$ confidence interval for the common variance.