2020

STATISTICS — HONOURS

Paper : CC-2

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any ten*** questions : 1×10

(a) A couple decides to have children until either they have both a boy and a girl or they have three children. Write down the sample space.

(b) We stop three people at random and ask the day of the week on which they were born. What is the probability that they are born on 3 days of the week in succession?

(c) Let A and B be two events such that $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.1$. Find the probability that A or B occurs, but not both.

(d) An experiment has only two outcomes. The first has probability p to occur, the second probability p^2. What is p?

(e) A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find the probability that a number greater than 3 occurs on a single roll of the die.

(f) Give an example of a sequence of events A_i in $\Omega = [0, 1]$, $i = 1, 2, ...$ such that $\bigcup_{i=1}^{\infty} A_i = [0,1)$ and $\bigcap_{i=1}^{\infty} A_i = \{0\}$.

(g) Give an example of a probability density function (pdf) which is not bounded.

(h) Give an example of a function g such that if μ_e is the median of X then $g(\mu_e)$ is not the median of $Y = g(X)$.

(i) Give an example of a continuous random variable X for which median of X is $ln\ 2$.

(j) Find the expected value of the discrete random variable X having the probability mass function $f(x) = |x-2| / 7$ for $x = -1, 0, 1, 3$.

(k) For a random variable X with pdf $f(x) = \frac{3}{4}x(2-x), 0 \le x \le 2$, what is the value of the third order central moment?

(l) Find the third quartile of a random variable X with pdf $f(x) = 2(1-x), 0 \le x \le 1$.

(m) Give an example of two random variables X and Y for which $P[X = x] = P[Y = x]$ for all x, but X and Y are not equal.

(n) Suppose $P(A) = P(B) = 0.9$. Give a useful lower bound on $P(B \mid A)$.

(o) What is the smallest value of K in Chebyshev's inequality for which the probability that a random variable with mean μ and variance σ^2 will take on a value between $\mu - K\sigma$ and $\mu + K\sigma$ is at least 0.95?

2. Answer ***any four*** questions : 5×4

(a) Suppose you are given a biased coin. Discuss how you will find the probability of getting a 'head' in a single toss of the coin.

(b) An experiment consists of rolling a die until a 3 appears. Describe the sample space and find the probability that the 3 appears not later than the k^{th} roll of the die.

(c) Suppose that the birthday of each of three people is equally likely to be any one of the 365 days of the year independently of others. Let B_{ij} denote the event that person i has the same birthday as person j, where $i, j = 1, 2, 3$.

(i) Are the events B_{12}, B_{23} and B_{31} pairwise independent?

(ii) Are the events B_{12}, B_{23} and B_{31} mutually independent?

(d) Suppose a fair die is rolled twice and that X is the absolute value of the difference of the two rolls. Find the probability mass function (pmf) and the cumulative distribution function (cdf) of X. Also find a median of X.

(e) An urn contains four red and four green balls, which are taken out without replacement, one at a time, at random. Let X be the first draw at which a green ball is taken out. Find the pmf and the expected value of X.

(f) Let X be a non-negative integer valued random variable.

Show that $E(X^2) - E(X) = 2\sum_{n=1}^{\infty} n \,.\, P(X > n)$.

3. Answer ***any two*** questions : 10×2

(a) Give the axiomatic definition of probability. Construct a probability function in case the sample space is finite. Also verify that the function is really a probability function. Derive 'classical definition' as a special case of this probability function.

(b) An urn contains n black and n red balls. Two balls are removed from the urn together at random.

(i) Write down the sample space.

(ii) What is the probability of drawing two balls of different colours?

(iii) Find the probability p_n that the balls are of the same colour, and evaluate $\lim_{n \to \infty} p_n$.

(iv) Half of the balls are removed and placed in a box. One of those remaining in the urn is drawn at random. What is the probability that it is red?

(c) An urn contains N tickets numbered 1, 2,..., N. A sample of size $2n + 1$ is drawn without replacement and let M be the sample median. Find the pmf of M. Also find $E(M)$ and var(M).