2020

STATISTICS — HONOURS

Paper : CC-11

(Statistical Inference-II)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any ten*** questions : 1×10

(a) Distinguish between an Estimator and an Estimate.

(b) Give a real life example of a two-sided alternative hypothesis.

(c) State two properties of Likelihood ratio test.

(d) State one use of Pearsonian χ^2-statistic.

(e) Explain the meaning of the first letter U in UMVUE.

(f) Let $(X_1, X_2, ..., X_n)$ be a random sample from normal (μ, σ^2)-distribution, where both μ and σ are unknown.

Define $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$, where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. We know that s^2 is unbiased for σ^2. Show that s is NOT unbiased for σ.

(g) Define a uniformly most powerful unbiased test.

(h) State one advantage of Rao–Blackwell theorem.

(i) Let $(X_1, X_2, ..., X_n)$ be a random sample from normal (μ, σ^2) distribution, where both μ and σ are unknown. Suggest two sufficient statistics for μ.

(j) Give an example of an unbiased estimator, involving all the members of a random sample $(X_1, X_2, ..., X_n)$, which is not consistent.

(k) State one merit and one demerit of Likelihood ratio test.

(l) Give an example where the asymptotic normality of a sequence of random variables can be proved using Lindeberg–Levy Central Limit Theorem.

(m) What does the term "accurate" signify in a UMA confidence set?

(n) What do you mean by confidence coefficient of a confidence interval?

(o) Explain the term 'test function'.

2. Answer ***any four*** questions : 5×4

(a) If $X_n \xrightarrow{P} 1$, then using the definition of convergence in probability verify whether $X_n^2 \xrightarrow{P} 1$.

(b) Give an example of a test for which Probability [Type-I error] + Probability [Type-II error] is greater than 1.2. Can it be a most powerful test? Justify your answer.

(c) Let $(X_1, X_2, ..., X_n)$ be a random sample from uniform $(\theta, 1)$-distribution, $\theta < 1$. Define $X_{(1)} = \min\{X_1, X_2, ..., X_n\}$. Find MSE and bias of $X_{(1)}$.

(d) Let $(X_1, X_2, X_3, ..., X_n)$ be a random sample from exponential $\left(\text{Mean} = \frac{1}{\beta}\right), (\beta > 0)$ distribution. If $g(\beta) = P(X_1 > 1)$, then find the asymptotic distribution of $g(\bar{X}_n)$, where $\bar{X}_n$ is the sample mean of $X_1, X_2, X_3 ..., X_n$.

(e) Consider any discrete distribution which is a member of one parameter exponential family. Derive likelihood ratio test for a simple null hypothesis against one-sided alternative based on a random sample of size n from the distribution chosen by you.

(f) Write a note on Rao-Cramer inequality.

3. Answer ***any two*** questions : 10×2

(a) State Neyman-Pearson lemma and prove its sufficiency part. Suppose $X \sim$ exponential (Mean = θ) and $Y \sim N(0, \text{variance} = \theta/2)$, $\theta > 0$ and X & Y are independent. Based on X and Y, find a uniformly most powerful test at level $\alpha = 0.05$ for testing $H_0 : \theta = 1$ against $H_1 : \theta < 1$ as explicitly as possible.

(b) (i) Define an unbiased estimator. Suppose $X \sim$ Uniform $[-1, \theta]$, $\theta > 0$. Define $Y = 1$ if $X > 0$ and $Y = 0$, otherwise. Is it possible to find real numbers a and b such that $a + bY$ is an unbiased estimator of θ? Is it possible to find real numbers a and b such that $a + bY$ is an unbiased estimator of $\frac{\theta - 1}{\theta + 1}$?

(ii) Explain the concept of maximum likelihood estimator (MLE). Let $(X_1, X_2, X_3, ..., X_n)$ be a random sample from exponential (Mean = 2θ) distribution. Find (with justification) MLE of $P(X_3 > 3)$.

(c) (i) Let $\{X_n\}$ be a sequence of independent random variables such that $E(X_n) = \mu$ and $\text{Var}(X_n) \le \frac{2021}{n^{1.202}}$ for all n = 1, 2, 3... . Verify whether the Weak law of large numbers holds.

(ii) Discuss a large sample test for testing the equality of two Poisson means. If the null hypothesis is accepted, describe a method for constructing a 95% confidence interval of the common value.