2020

STATISTICS — HONOURS

Paper : DSE-B-1

(Stochastic Processes and Queuing Theory)

Full Marks : 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Choose the correct alternative (***any ten***) [only the first ***ten*** answers will be considered for evaluation] : 1×10

(a) For a non-constant weakly stationary process,

(i) both the mean and variance functions are constants

(ii) the mean functions is an increasing linear function and the variance function is constant

(iii) the mean function is constant and the variance function is an increasing linear function

(iv) the mean and variance are both increasing linear functions.

(b) For a discrete-time strictly stationary process $(X_n)_{n\geq 0}$, the joint distribution of

$\left(X_m, X_{m+n_1}, X_{m+n_1+n_2}\right)$ where $m, n_1, n_2 \in \mathbb{N}$, is free of m

(i) for every choice of n_1 and n_2 (ii) if and only if $n_1 < n_2$

(iii) if and only if $n_1 = n_2$ (iv) if and only if $n_1 > n_2$

(c) Suppose two states x and y in a Markov chain communicate. Which of the following is FASLE?

(i) Neither can be absorbing (ii) If either is transient, so must the other be

(iii) Their periods must be equal (iv) Their mean return times must be equal.

(d) Suppose in a finite state Markov chain, whenever $x \rightsquigarrow y$ for two states x and y of the chain, always $y \rightsquigarrow x$ too. Then

(i) the chain must be irreducible

(ii) the chain cannot have any transient state

(iii) the chain cannot have any absorbing state

(iv) the chain cannot have any positive recurrent state.

(e) A finite state Markov chain cannot have any

(i) absorbing state (ii) transient state

(iii) null recurrent state (iv) positive recurrent state.

(f) The transition probability matrix of a certain Markov chain is doubly stochastic. Which of the following is FALSE?

(i) The chain must have a unique stationary distribution

(ii) The chain must have at least one transient state

(iii) The chain must be ergodic

(iv) All states must have equal mean return time.

(g) Suppose for an irreducible discrete-time birth and death chain on $\{0, 1, 2, ...\}$, p_x and q_x denote respectively $p_{x, x+1}$ and $p_{x, x-1}$ for $x \geq 1$, where the symbols have their usual meanings. Then,

(i) if $p_x < q_x$ $\forall x$, the chain must be transient

(ii) if $p_x < q_x$ $\forall x$, the chain must be recurrent

(iii) if $p_x > q_x$ $\forall x$, the chain must be transient

(iv) if $p_x > q_x$ $\forall x$, the chain must be recurrent.

(h) If a branching chain has a binomial $B\left(2, \frac{1}{2}\right)$ offspring distribution, then its extinction probability is

(i) 0 (ii) $\frac{1}{4}$

(iii) $\frac{1}{2}$ (iv) 1

(i) For an irreducible Markov chain on four states $\{a, b, c, d\}$, the mean return times m_x for $x = a, b, c$ are given as $m_a = m_b = m_c = 6$. Then m_d equals

(i) 2 (ii) 4

(iii) 6 (iv) 18

(j) A type of radiation is emitted by the source as a Poisson process with rate of 3 per minute and each is captured by a certain counter with probability ½ each, independently. Then the expected time for the 100th emission to be captured is

(i) less than 1 hour (ii) between 1 hour and 2 hours

(iii) between 2 hours and 24 hours (iv) more than 24 hours.

(k) For a pure birth process, the forward equations with standard notation are

(i) $p'_{xy}(t) = \lambda_x p_{x,y}(t) - \lambda_{x+1} p_{x+1,y}(t)$ (ii) $p'_{xy}(t) = \lambda_y p_{x,y-1}(t) - \lambda_{y+1} p_{xy}(t)$

(iii) $p'_{xy}(t) = \lambda_{y-1} p_{x,y-1}(t) - \lambda_y p_{xy}(t)$ (iv) $p'_{xy}(t) = \lambda_{x-1} p_{x-1,y}(t) - \lambda_x p_{xy}(t)$

(l) The production of a certain product in a large manufacturing plant follows a Poisson process with rate 3 per minute. If in the first hour the number of products produced was 300, then the expected total number produced after the first hour and a half is

(i) 450 (ii) 480

(iii) 600 (iv) 900

(m) Suppose for a continuous-time birth and death chain on $\{0, 1, 2, ...\}$, the death rate q_x at state x is x and the bith rate $p_x = x + k$ for a non-negative integer k. Then the chain is transient if and only if

(i) $k = 0$ (ii) $k \leq 1$

(iii) $k = 1$ (iv) $k > 1$

(n) Suppose customers arrive at a service centre with a single server following a Poisson process with rate λ and the service time $T > 0$ is fixed. Then the queue will be transient if

(i) $\lambda < T^{-1}$ (ii) $\lambda > T^{-1}$

(iii) $\lambda < T$ (iv) $\lambda > T$

(o) If customers arrive in a queue following a Poisson process with rate 12 per hour and the service times are exponential with mean 20 minutes then a necessary and sufficient condition the number k of servers for the queue length to not become indefinitely large with certainty, is

(i) none, any $k \geq 1$ will do (ii) $k \geq 2$

(iii) $k > 2$ (iv) $k > 4$

2. Write short notes ***fully in your own words*** on ***any four*** of the following : 5×4

(a) Weak and strong stationarity in discrete time.

(b) Distribution of number of visits to a transient state in a Markov chain.

(c) Aperiodicity as a class property in Markov chains, with proof.

(d) Distribution of increments of a Poisson process.

(e) Transition probabilities for a pure birth process.

(f) Queue disciplines.

3. Write essays ***fully in your own words*** on ***any two*** of the following : 10×2

(a) Definition, existence and uniqueness of stationary distributions of a Markov chain.

(b) Equivalence of sets of postulates of a Poisson process.

(c) Steady state distribution and performance measures for an M/M/1 queue.