

**2020**

**ADVANCED BUSINESS MATHEMATICS — HONOURS**

**Sixth Paper**

**(A-32-A)**

**Full Marks : 50**

*The figures in the margin indicate full marks.  
Candidates are required to give their answers in their own words  
as far as practicable.*

**Group - A**

1. Answer **any four** questions :

5×4

(a) Find the domain of definition of the function  $\log(x^2 - 5x + 6)$

**Or,**

If  $f(x) = [x - 1] - x$ , then find  $f(0)$  and  $f(1)$ .

(b) Draw the graph of the function  $f(x) = |x|$ .

(c) If  $f(x) = \frac{x-1}{x+1}$ , then show that  $\frac{f(a)-f(b)}{1+f(a)f(b)} = \frac{a-b}{1+ab}$

**Or,**

If  $3f(x) - 2f(-x) = 10x-1$ , find  $f(x)$  and hence find  $f(2-3x)$ .

(d) Find  $\lim_{x \rightarrow -4} \left[ \frac{1}{x+4} + \frac{8}{x^2-16} \right]$

(e) Differentiate  $e^{\sqrt{x}}$  with respect to  $x$ .

**Or,**

If  $x^2 + y^2 = 2\log(x+y)$ , find  $\frac{dy}{dx}$ .

**Please Turn Over**

**Group - B**

2. Answer *any two* questions : 10×2

(a) Evaluate :

(i)  $\lim_{x \rightarrow 0} \frac{e^{ax} - e^{bx} + kx}{x}$

(ii)  $\lim_{x \rightarrow \infty} \frac{4x^4 - 3x + 2}{5x^4 + 2x^2 + 3}$  5+5

(b) If  $x = \frac{1-t}{1+t}$  and  $y = \frac{2t}{1+t}$ , then show that  $\frac{d^2y}{dx^2} = 0$  10

(c) Show that the maximum value of  $x + \frac{1}{x}$  is less than its minimum value. 10

**Or,**

The sum of two positive numbers is 24. Find the two numbers if the sum of their squares is minimum. 10

(d) Prove that (without direct expansion) 10

$$\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix} = (x-a)(x-b)(x+a+b)$$

**Or,**

Solve by matrix inversion method : 10

$$x + y + z = 3; \quad x - y + z = 1; \quad x + y - z = 1$$

(e) If  $A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$ , find  $A^2 - A$ . 10

**Or,**

Express the following in a single matrix : 10

$$\begin{bmatrix} 3 & 2 & 5 \\ 2 & -4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 7 & 29 \\ -6 & 8 \end{bmatrix}$$

**Group - C**

3. Answer *any one* question : 10×1

(a) Evaluate

(i)  $\int \frac{2x+3}{3x+2} dx$       (ii)  $\int \frac{2x dx}{2+x-x^2}$  10

(b) A number is chosen at random from the first 50 positive integers. Find the probability that the chosen number is divisible by 3 or 5. 10

(c) If 10 consecutive days are chosen at random, what is the probability that two of them will be Saturdays? 10

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