2021

MATHEMATICS — GENERAL

Paper : SEC-B-2

(Boolean Algebra)

Full Marks : 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

(Marks : 20)

1. Choose the correct alternative and justify your answer : 2×10

(a) An order relation is

(i) reflexive, symmetric and transitive

(ii) reflexive, antisymmetric and transitive

(iii) reflexive, symmetric and antisymmetric

(iv) antisymmetric, symmetric and transitive.

(b) It is false that

(i) $(\mathbb{Z}, \leq)$ is a chain (ii) $(\mathbb{Q}, \leq)$ is a chain

(iii) $(\mathbb{R}, \leq)$ is a chain (iv) $(\mathbb{C}, \leq)$ is a chain.

(c) It is true that in a order set

(i) there may be more than one maximal elements but no greatest element.

(ii) there is maximal element as well as greatest element.

(iii) there is always a greatest element.

(iv) there is always a smallest element.

(d) Let L and M be two lattices and let $f : L \to M$ be a homomorphism. Then f is

(i) only join-homomorphism (ii) only meet-homomorphism

(iii) only order-homomorphism (iv) both join-homomorphism and meet-homomorphism.

(e) Let $(B, +, . , ')$ be a Boolean algebra and $a, b \in B$. Then

(i) $a + b = b + a$, but $a.b \neq b.a$ (ii) $a + b \neq b + a$, but $a.b = b.a$

(iii) $a + b = b + a$ and $a.b = b.a$ (iv) $a + b \neq b + a$ and $a.b \neq b.a$.

(f) Let $(L, \wedge, \vee)$ be a lattice where $\wedge$ denotes meet operator and $\vee$ denotes join operator. For any three elements a, b and c absorption law is given by

(i) $a \wedge (b \wedge c) = a$ (ii) $a \wedge (a \wedge c) = a$

(iii) $a \wedge (a \vee b) = a$ (iv) $a \vee (a \vee c) = a$.

(g) Dual of the statement $(a \wedge b) \vee c = (b \vee c) \wedge (c \vee a)$ is given by

(i) $(a \vee b) \vee c = (b \vee c) \wedge (c \vee a)$ (ii) $(a \wedge b) \wedge c = (b \vee c) \vee (c \vee a)$

(iii) $(a \vee b) \wedge c = (b \wedge c) \vee (c \wedge a)$ (iv) $(a \wedge b) \vee c = (b \wedge c) \wedge (c \vee a)$.

(h) Let $X = \{1, 2, 3\}$ be a set. Then $P(X)$, the power set of X form a Boolean algebra under the set theoretical operation of union, intersection and complementation. The complement of the element $\{2\}$ in $P(X)$ is given by

(i) $\{3\}$ (ii) $\{1\}$

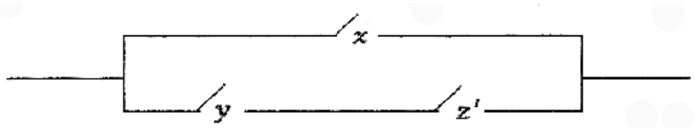
(iii) $\{1, 3\}$ (iv) $\{1, 2, 3\}$.

(i) In Boolean algebra which of the following equality is true?

(i) $(a + b)' = a' + b'$ (ii) $(a + b)' = a' * b'$

(iii) $(a + b)' = a + b$ (iv) $(a + b)' = a + b'$.

(j)



Boolean expression corresponding to the above circuit is written as

(i) xyz' (ii) $xy + z'$

(iii) $x + yz'$ (iv) $xz' + y$.

Group - B

(Marks : 60)

Answer ***any six*** questions.

2. (a) When a relation on a non empty set is called a partial order relation on a non empty set? If a relation R is defined on the set Z of all integers by $a \leq b \leftrightarrow a^2 = b^2$. Is R a partial order?

(b) Let $\mathbb{N}$ denotes the set of natural numbers. If a relation $\leq$ on $\mathbb{N}$ is defined as $a \leq b \leftrightarrow a$ divides b then show that it is a partial order relation on $\mathbb{N}$.

(c) Write the main differences between partial order relation and equivalence relation on a non empty set. (2+2)+4+2

3. (a) Prove that a lattice $(P, \leq)$ is distributive if and only if $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ for all $a, b, c \in L$.

(b) State and prove De Morgan's Laws in Boolean Algebra. 5+5

4. (a) Let $(P, \leq)$ be a partially ordered set. When $(P, \leq)$ is called a lattice ordered set?

(b) When a lattice is called a complete lattice?

(c) Let $P(X)=\{\phi, \{1\}, \{2\}, \{1, 2\}\}$ where $X = \{1, 2\}$ and a order relation $\leq$ is defined on $P(X)$ as $A \leq B \leftrightarrow A \subseteq B$. Show that $(P(X), \leq)$ is a lattice. 3+3+4

5. (a) When a lattice is called a distributive lattice?

(b) Show that every distributive lattice is a modular lattice.

(c) Let $S = \{1, 2, 3, 4, 12\}$ and a partial order relation $\leq$ is defined on S as $a \leq b \leftrightarrow a$ divides b. Then $(S, \leq)$ forms a lattice. Is this a distributive lattice? 3+4+3

6. (a) When a non empty set is said to form a Boolean algebra with respect to two binary operation + and $*$ and one unary operation $'$? Give an example of a Boolean algebra.

(b) Show that if a and b are any two elements in Boolean algebra B then prove that

(i) $a + a = a$, (ii) $a + (a * b) = a$. 4+(3+3)

7. (a) What is Boolean polynomial? Give an example of Boolean polynomial.

(b) Use the method of *Karnaugh map* to find the minimal form of the following Boolean expression :

$$E = xyz + xyz' + x'yz' + x'y'z' + x'y'z$$ (2+2)+6

8. (a) Let L be a lattice and $a, b, c, d \in L$. Then prove that $a \leq b$ and $c \leq d \Rightarrow a \vee c \leq b \vee d$.

(b) Prove that every finite lattice is complete lattice. Is the converse true? 5+5

9. (a) Let $(L, \wedge, \vee)$ be a distributive lattice and $a, b, c \in L$.
Prove that $a \wedge c = b \wedge c$ and $a \vee c = b \vee c \Rightarrow b = a$.

(b) A committee consisting of three members approves any proposal by majority vote. Each member can approve a proposal by pressing a button attached to their seats. Design as simple a circuit as you can which allow current to pass when and only when a proposal is approved. 5+5

10. (a) Let B be the set of all positive integers which are divisors of 70. For $a, b \in B$, let $a + b =$ *l.c.m.* of a, b; $a.b =$ *h.c.f.* of a, b and $a' = \frac{70}{a}$. Prove that $(B, +, \cdot, ')$ is a Boolean algebra.

(b) Let $(B, +, \cdot, ')$ be a Boolean algebra and $a, b, c \in B$.
Prove that $(a + b)\cdot(b + c)\cdot(c + a) = (a\cdot b) + (b\cdot c) + (c\cdot a)$. 5+5

11. (a) Construct a truth table for the Boolean expression : $xy' + y(x' + z)$.

(b) Find a switching circuit which realizes the switching function f given by the following switching table : 5+5

x	y	z	$f(x, y, z)$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1