2021

MATHEMATICS — HONOURS

Paper : CC-2

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Choose the correct alternative with proper justification, 1 mark for correct answer and 1 mark for justification. 2×10

(a) Let $\mathbb{N}$ be the set of natural numbers and a relation '$\leq$' on $\mathbb{N}$ is defined by "$a \leq b$ if and only if a is less than or equal to b". Then $(\mathbb{N}, \leq)$

(i) is a poset but not linearly ordered set

(ii) poset as well as linearly ordered set

(iii) linearly ordered set but not poset

(iv) none of these.

(b) The remainder when 2^{44} is divided by 89 is

(i) 1 (ii) 3 (iii) 6 (iv) 11

(c) If $f : \mathbb{R} \setminus \{1, -1\} \to \mathbb{R}$ be a function defined by $f(x) = \dfrac{2x-1}{x^2-1}$, then f is

(i) bijective (ii) neither injective nor surjective

(iii) injective but not surjective (iv) surjective but not injective.

(d) Number of partitions on a set $S = \{a, b, c, d\}$ is

(i) $2^{16} - 1$ (ii) 2^4 (iii) 2^8 (iv) 2^{16}.

(e) The values of i^i form a/an

(i) HP (ii) AP (iii) GP (iv) none of these.

(f) The equation $x^5 + x^3 - x^2 + x - 1 = 0$ has

(i) all real roots (ii) two negative real roots

(iii) two positive and two negative real roots (iv) at least two imaginary roots.

(g) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then the value of $\sum \frac{1}{\alpha^2 - \beta\gamma}$ is

(i) $\frac{3}{q}$ (ii) $-\frac{3}{q}$ (iii) $\frac{1}{q}$ (iv) $-\frac{1}{q}$.

(h) Let $f : \mathbb{N} \to \mathbb{N}$ be defined by $f(n) = {}^{n+1}C_n$, then

(i) f is injective but not surjective (ii) f is not injective but surjective

(iii) f is injective and surjective (iv) f is neither injective nor surjective.

(i) The rank of the matrix $\begin{pmatrix} 1 & 0 & 1 \\ \alpha & 1 & \beta \\ 0 & 0 & 0 \end{pmatrix}$; $\alpha, \beta \in \mathbb{R}$,

(i) depends on the value of α and β (ii) depends on the value of α

(iii) depends on the value of β (iv) independent of the value of α and β.

(j) The system of linear equation

$$x + 2y + z = 1$$
$$2x + y + 3z = b$$
$$x - 4y + 3z = b + 1$$

has infinitely many solutions if

(i) $b = 4$ (ii) $b \neq 4$ (iii) $b = -4$ (iv) $b \neq -4$.

2. Answer ***any four*** questions :

(a) If real quantities x, y; u, v are connected by the equation $\cosh(x + iy) = \cot(u + iv)$, then show that

$$\frac{\sinh 2v}{\sin 2u} = -\tanh x \tan y$$ 5

(b) Solve the equation $4x^4 + 20x^3 + 35x^2 + 24x + 6 = 0$ whose roots are in A.P. 5

(c) Solve the equation $x^3 - 12x + 8 = 0$ by Cardon's Method. 5

(d) Find the general solution of the linear difference equation $u_{x+2} - 3u_{x+1} - 4u_x = 2^x$. 5

(e) (i) If $2\cos\theta = t$, prove that $\frac{1 + \cos 7\theta}{1 + \cos\theta} = \left(t^3 - t^2 - 2t + 1\right)^2$.

(ii) Prove that $\sin 7\theta = 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64\sin^7\theta$. 2+3

(f) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then form the equation whose roots are $\alpha - \frac{1}{\beta\gamma}, \beta - \frac{1}{\gamma\alpha}, \gamma - \frac{1}{\alpha\beta}$. 5

(g) (i) Prove that $\frac{1}{2\sqrt{n+1}} < \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdots\frac{2n-1}{2n} < \frac{1}{\sqrt{2n+1}}$.

(ii) If x, y, z are real and not all equal, show that $x^3 + y^3 + z^3 > 3xyz$, if $x + y + z > 0$. 3+2

3. Answer ***any four*** questions :

(a) (i) Give an example of a binary relation which is transitive but neither reflexive nor symmetric.

(ii) Show that the number of different reflexive relations on a set of n elements is 2^{n^2-n}. 3+2

(b) Let $f : A \to B$ be a mapping. Prove that f is invertible if and only if f is a bijection. 5

(c) If $d = gcd\ (a, m)$, then prove that $ax \equiv ay \pmod{m}$ if and only if $x \equiv y\left(\bmod \frac{m}{d}\right)$. 5

(d) State and prove Chinese remainder theorem. 5

(e) (i) Let R_1 and R_2 be equivalence relations on a set S such that $R_1 \circ R_2 = R_2 \circ R_1$. Prove that $R_1 \circ R_2$ is an equivalence relation.

(ii) Let $(A, \leq_1)$ and $(B, \leq_2)$ be two posets. Prove that $(A \times B, \leq)$ is a poset, where $(a, b) \leq (c, d) \Leftrightarrow a \leq_1 c$ and $b \leq_2 d$. 2+3

(f) (i) Let n be a natural number, and let $f : \{i \in \mathbb{N} : 1 \leq i \leq n\} \to \mathbb{N}$ be a function. Show that there exists a natural number M such that $f(i) \leq M$, for all $1 \leq i \leq n$.

(ii) A mapping f is defined by $f : \mathbb{N} \to \mathbb{N}$, $f(n) = \left[\frac{n+1}{2}\right], n \in \mathbb{N}$, show that f is surjective but not injective. 3+2

(g) (i) Prove or disprove :
Let $f : X \to Y$ be a function. Then f is injective if and only if $f(A \cap B) = f(A) \cap f(B)$ for all non-empty subsets A and B of X.

(ii) Let A be a non-empty set and ρ be a relation on A. Let B denote the set of all ρ-equivalent classes. Prove that there exists a surjective function from A onto B. 3+2

4. Answer ***any one*** question :

(a) Reduce the given matrix to its row-echelon form and determine the rank of the matrix

$$\begin{pmatrix} 0 & 2 & 4 & 2 & 2 \\ 4 & 4 & 4 & 8 & 0 \\ 8 & 2 & 0 & 10 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$$ 5

(b) Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9$; $7x + 3y - 2z = 8$ and $2x + 3y + \lambda z = \mu$ have

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions. 5
