2021

MATHEMATICS — HONOURS

Paper : SEC-B-1

(Mathematical Logic)

Full Marks : 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Choose the correct alternative. Each question carries **2** marks, **1** mark for correct option and **1** mark for its justification. 2×10

(a) Which one of the following well formed formula is not a tautology?

(i) $P \vee \neg P$ (ii) $P \rightarrow \neg\neg P$

(iii) $\neg(P \rightarrow P)$ (iv) $Q \rightarrow P \vee Q$.

(b) $P \rightarrow (Q \wedge \neg Q)$ is logically equivalent to

(i) P (ii) $\neg P$

(iii) Q (iv) $\neg Q$.

(c) Which of the following is not a well formed propositional formula?

(i) $\left(\neg\left(p \rightarrow \left(q \wedge p\right)\right)\right)$ (ii) $\left(\neg\left(p \rightarrow \left(q = p\right)\right)\right)$

(iii) $(p \rightarrow q) \vee (p \rightarrow \neg q)$ (iv) $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$.

(d) Disjunctive normal form of the formula $p \leftrightarrow q$ is

(i) $(p \wedge q) \vee (p \wedge \neg q)$ (ii) $(p \wedge \neg q) \vee (p \wedge q)$

(iii) $(\neg p \wedge q) \vee (p \wedge q)$ (iv) $(p \wedge q) \vee (\neg p \wedge \neg q)$.

(e) A counter example to the universally quantified statement $\forall x \forall y \left(x^2 \neq y^3\right)$ where the domain for all variables is the set of integers, is

(i) $x = 8$, $y = 4$ (ii) $x = 3$, $y = 4$

(iii) $x = 8$, $y = 9$ (iv) $x = 4$, $y = 2$.

(f) Let $Q(x, y)$ be the statement "$x + y = x - y$". If the domain for both variables consists of all integers, which one of the following has the truth value TRUE?

(i) $\forall y\ Q(1, y)$ (ii) $\exists x\ Q(x, 2)$

(iii) $\forall y\ \exists x\ Q(x, y)$ (iv) $\exists y\ \forall x\ Q(x, y)$.

(g) The number of different non-equivalent statement formulas containing n statement letters is

(i) 2^{n^2} (ii) 2^n

(iii) 2^{2^n} (iv) 2^{2n}.

(h) An one element adequate system of connective is

(i) $\{\wedge\}$ (ii) $\{\vee\}$

(iii) $\{\,|\,\}$ (iv) $\{\sim\}$.

(i) Let $p(x)$ be a predicate on a non-empty set D. Then $\sim\forall\, x\, p(x) \equiv$

(i) $\exists\, x \sim p(x)$ (ii) $\sim\exists\, x\, p(x)$

(iii) $\forall\, x \sim p(x)$ (iv) $\sim\exists\, x \sim p(x)$.

(j) The inverse of the statement formula $(\sim r \rightarrow s)$ is

(i) $(r \rightarrow \sim s)$ (ii) $(\sim r \rightarrow \sim s)$

(iii) $(s \rightarrow r)$ (iv) $(s \rightarrow \sim r)$.

Unit - I

2. Answer ***any two*** questions :

(a) (i) State the converse, contrapositive and inverse of the following conditional statement :

If it rains, there is cloud in the sky.

(ii) Let P and Q be the propositions "*Swimming at the New Jersey shore is allowed*" and "*Sharks have been spotted near the shore*", respectively. Express each of these compound propositions as an English sentence.

(A) $\neg P \vee Q$ (B) $P \leftrightarrow \neg Q$. 3+2

(b) Find the truth value of the statement "if x is an odd integer, then x^2 is an odd integer". Write down the converse and contrapositive statement of the above statement with its truth value. 1+(2+2)

(c) Find the truth table of the statement formula : $\big((\sim p \rightarrow q) \wedge (r \rightarrow p)\big) \leftrightarrow (\sim q \vee r)$. 5

(d) (i) Remove as many brackets as possible from the following statement formula :

$$\Big(\big(\neg(P \vee R)\big) \vee \big(\neg(Q \leftrightarrow P)\big)\Big).$$

(ii) Find a propositional formula that represents the following proposition :

If I win the lottery or pass the examination, then I am happy. 3+2

Unit - II

3. Answer ***any six*** questions :

(a) (i) Construct a truth table for the compound proposition

$$(P \leftrightarrow Q) \vee (\neg Q \leftrightarrow R).$$

(ii) Verify the following logical equivalence of the following statement by truth table.

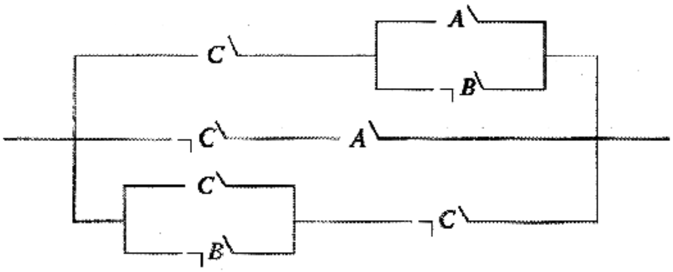
$$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P).$$ 2+3

(b) (i) When is a collection of propositional connectives said to form an adequate system of connectives?

(ii) Examine whether the set $\{\vee, \rightarrow\}$ is an adequate system of connectives.

(iii) Show that the binary connective $\downarrow$ (*NOR*) alone forms an adequate system. 1+2+2

(c) Write the equivalent statement form representing the following circuit. Find the simplest equivalent circuit. 2+3



(d) Suppose A and B are two statement forms in Propositional logic.

(i) When is A said to logically imply B?

(ii) When are A and B said to be logically equivalent?

(iii) Show that $\neg P \rightarrow (Q \rightarrow R)$ and $Q \rightarrow (P \vee R)$ are logically equivalent. 1+1+3

(e) (i) Suppose S is a set of propositional formulas and F be a propositional formula. When is F said to be a logical consequence of the formulas is S?

(ii) Show that $\left(P \rightarrow (Q \vee R)\right)$ is a logical consequence of $(P \rightarrow Q)$.

(iii) When is a statement form said to be in *disjunctive normal form*? 2+2+1

(f) (i) Determine whether the following statement is true or false. Justify your answer.

If the formula $\left(P \vee (P \wedge Q)\right)$ *is true, then P is true.*

(ii) Determine whether the following propositional formula is satisfiable :

$\left(P \rightarrow (Q \rightarrow R)\right) \wedge (Q \wedge \neg R)$. 3+2

(g) Show that the collection $\{\sim, \wedge, \vee\}$ is an adequate system of connectives. 5

(h) Determine whether the given arguments are logically correct : If Samar did not meet Sajal last night, then either Sajal was the murderer or Samar is lying. If Sajal was not the murderer, then Samar did not meet Sajal last night and the murder took place after midnight. If the murder took place after midnight, then either Sajal was the murderer or Samar is not lying. Hence Sajal was the murderer. 5

(i) State Deduction Theorem in Propositional logic.

Show that $\{B \rightarrow C, C \rightarrow D\} \vdash B \rightarrow D$ where B, C, D are well formed formulas in Propositional logic. 1+4

(j) Show that the well formed formula $B \rightarrow \left(C \rightarrow (B \wedge C)\right)$ is a theorem in Propositional logic, where B, C are well formed formulas in Propositional logic. 5

Please Turn Over

Unit - III

4. Answer ***any four*** questions :

(a) (i) Translate the following statement into logical expression using predicates, quantifiers and logical connectives :

At least one of your friends is perfect.

(ii) Determine the truth values of the following statements :

(A) $(\forall x \in \mathbb{R})(x^2 \geq x)$, (B) $(\forall x \in \mathbb{Z})(x^2 \geq x)$.

(iii) Find the negation of the following predicate :

$(\forall x \in D)(x + 3 < 9)$ where $D = \{1, 2, 3, 4\}$. 2+2+1

(b) (i) Transform the formula $\forall x p(x) \vee \exists y\, q(y)$ to Prenex normal form.

(ii) Find the truth set of the following predicate :

$(\forall y \in D)(x^2 + y < 12)$ where domain of x is the set of positive integers. 3+2

(c) (i) Suppose $p(x)$ and $q(x)$ are two predicates defined on a domain D. Show that $\forall x(p(x) \wedge q(x))$ and $\forall x p(x) \wedge \forall x q(x)$ are logically equivalent.

(ii) Find a counter example (if exists) for the statement :

$(\forall x \in D)(x$ *is prime*$)$ where $D = \{1, 3, 5, 7, 11\}$. 4+1

(d) (i) State the **completeness theorem** of Predicate logic.

(ii) Translate the following statement into simple English :

$\forall x((x \neq 0) \rightarrow \exists y(xy = 1))$

(iii) Symbolize the following sentence predicates, quantifiers and logical connectives :

A negative real number does not have a square root that is a real number. 1+2+2

(e) Define free and bound variable in a well formed formula. Find the free and bound variable(s) in the following well formed formula :

$(\forall x_1)\left(A_1^2(x_1, x_2) \rightarrow (\forall x_1) A_1^1(x_1)\right)$ 5

(f) Test the logical validity of the given arguments : All integers are rational numbers. All rational numbers are real numbers. 2 is an integer. Therefore, 2 is a real number. 5

(g) Prove that $\vdash (\forall x)(B \leftrightarrow C) \rightarrow ((\forall x)B \leftrightarrow (\forall x)C)$, where B, C are well formed formulas in Predicate logic. 5