

2021

MATHEMATICS — HONOURS

Paper : SEC-B-2

(Scientific Computing with SageMath/R)

Full Marks : 80

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

(Answer either Scientific Computing with SageMath or Scientific Computing with R)

[Scientific Computing with SageMath]

Notations and symbols have their usual meaning

Throughout the question paper sage code means code in SageMath.

1. Choose the correct alternative (Give justification whenever it is applicable) : 2×10
- (a) What is the correct code in Sage to evaluate the value of π correct up to 100 digits?
- | | |
|---------------------------------|--|
| (i) $n(\pi, 100)$ | (ii) $n(\text{pi}, \text{digits}=100)$ |
| (iii) $N(\pi, \text{prec}=100)$ | (iv) $N(\text{pi}, 100)$. |
- (b) What will be the result of the following Sage code: $\exp(11*8+5) - e^{(11*8+5)}$?
- | | |
|---------------------------------|----------------------------------|
| (i) 0 | (ii) error: “exp” is not defined |
| (iii) error: “e” is not defined | (iv) error: “e^” is not defined. |
- (c) What will be your code in Sage to find the natural logarithm of 100 in decimal approximation?
- | | |
|---------------------------|-------------------------|
| (i) $N(\log(100))$ | (ii) $n(\log(100,10))$ |
| (iii) $n(\log \exp(100))$ | (iv) $N(\log e(100))$. |
- (d) What will be your code in Sage, if you want all the square roots of 4?
- | | |
|---|---|
| (i) $\text{Sqrt}(4)$ | (ii) $\text{sqrt}(4)$ |
| (iii) $\text{sqrt}(4, \text{all}==\text{true})$ | (iv) $\text{sqrt}(4, \text{all}=\text{true})$. |
- (e) The output of the Sage code : $4*(10//4) + 10\%4 == 10, 3*3<3$ is
- | | |
|---------------------|----------------------|
| (i) (True, False) | (ii) (True, True) |
| (iii) (False, True) | (iv) (False, False). |

Please Turn Over

(f) Which of the following is NOT the correct option for a^{b^c} in sage?

(i) $a \wedge b \wedge c$

(ii) $a ** b ** c$

(iii) $a \wedge \wedge b \wedge \wedge c$

(iv) $a \wedge b ** c$.

(g) What will be the output of the following sage code?

```
a=4
```

```
b=2
```

```
print(a+b*2)
```

(i) 36

(ii) 10

(iii) 8

(iv) 12

(h) What will be the output of the following sage code?

```
a =[1, 3]
```

```
b=[10, 20, a]
```

```
print(b)
```

(i) $[10, 20, [1, 3]]$

(ii) $[10, 20, 1, 3]$

(iii) An error will occur

(iv) $[1, 3, 10, 20]$

(i) What is the correct code in Sage to plot the function $x^3 - x$ in $-2 < x < 2$?

(i) $\text{plot}(x^3 - x, -2, 2)$

(ii) $\text{plot}(x^3 - x, -2, 2)$

(iii) $\text{plot}(x^3 - x, \text{range}[-2, 2])$

(iv) $\text{plot}(x^3 - x; -2 2)$

(j) What will be the code in Sage to compute $\int_0^1 \frac{x}{x^2 + 1} dx$?

(i) $\text{integral}(x/(x^2 + 1), x, 0, 1)$

(ii) $\text{integration}(x/(x^2 + 1), x, 0, 1)$

(iii) $\text{integral}(x/(x^2 + 1), x; 0, 1)$

(iv) $\text{integrate}(x/(x^2 + 1), 0, 1)$

Section - I

Answer *any one* question.

2. (a) What are open source softwares? “Sage is a free and open source mathematical software.” — Explain in brief.
- (b) Write sage codes to find the difference between golden ratio and $\frac{f_{42}}{f_{41}}$ using 20 digits where f_n is the n th term of the Fibonacci sequence. (2+4)+4

3. (a) What will be the output of the following sage commands? Explain your answer :

$$y = 3; y = 3 * y + 1; y = 3 * y + 1; y$$

- (b) What will be the output of the following sage commands?

(i) $79/12$

(ii) $79/12.0$

(iii) $70//12$

- (c) Give the output of the following sage commands

$$10 \wedge (1/2), \sin(60)$$

$$4+(1+1+1)+(2+1)$$

Section - II

Answer *any one* question.

4. Write Sage code to plot the graphs of the following in one plot :

(a) graph of the function $f(x) = x^2 + 1$ in the range $(-2, 2)$.

(b) graph of the tangent line to the above curve : $f'(x) = 2x$ in the range $(-2, 2)$ with gridlines.

(c) one big dot at $(1, 2)$ with size = 40.

4+4+2

5. (a) Write sage code to plot the graph of the function given by the polar curve

$$r = 1 + 2 \cos 3\theta, 0 \leq \theta \leq 2\pi.$$

- (b) Write sage code to plot the graph of the following function :

$$f(x) = \begin{cases} 1+x, & -1 \leq x \leq 0 \\ 1+x^3, & 0 < x < 1 \end{cases}.$$

- (c) Write sage code to plot the function $f(x) = \frac{1}{x} - x$ in $[-1, 1]$. Write also the equation of the asymptote (if it exists).

3+3+4

Section - III

Answer *any one* question.

6. Write sage code to do the following where $f(x) = x^3 + x$

2×5

(a) find $f''(x)$.

(b) find $f'''(x)$.

(c) draw the graph of $f(x)$ where $-5 < x < 5$ with colour blue.

(d) draw the graph of $f''(x)$ where $-5 < x < 5$ with colour green.

(e) draw the graph of $f'''(x)$ where $-5 < x < 5$ with colour red.

(All the graphs should be in one plot).

Please Turn Over

7. Write codes in sage to do the following :

2×5

(a) Evaluate $\int x^n dx$

(b) Evaluate $\int \frac{\cos x}{\sqrt{1 + \sin x}} dx$

(c) Evaluate $\int_0^1 3x^2 dx$

(d) Evaluate $\int_0^{\infty} \frac{2}{\sqrt{\pi}} e^{-x^2} dx$

(e) Evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$ to display 10 digits.

Section - IV

Answer *any one* question.

8. (a) Without using inbuilt functions write a program in sage to determine the total number of primes less than x , print the list of such primes and get an output for $x = 98$.
- (b) Without using inbuilt function write a program in sage to determine factorial of 10. 5+5
9. (a) Without using inbuilt functions write a program in Sage to determine in decimal approximation the arithmetic mean and geometric mean of a list of numbers and get an output for the list 20, 15, 24, 31, 45, 17.
- (b) Write the Sage codes (without using inbuilt functions) to find and print the median of the numbers 11, 23, -34, 40, 50. 5+5

Section - V

Answer *any one* question.

10. (a) Define a matrix in sage whose rows are (1, -3, 4, 7), (3, 4, 7, 9), (3, 7, 0, 11), (1, 3, -4, 8). Give sage code to find the row reduced echelon form of the matrix.

(b) Write code in sage to find $\det A$ and A^{-1} where $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix}$.

- (c) Assuming the non-trivial solution exists, write sage commands to solve the following system of equations with matrices :

$$\begin{aligned} 3x - 4y + 5z &= 14 \\ x + y - 8z &= -5 \\ 2x + y + z &= 7. \end{aligned} \qquad 4+2+4$$

11. (a) Write sage code to solve the differential equation $\frac{dy}{dx} = \cos x$.

- (b) Write sage commands to solve the following Initial Value Problem and plot the solution :

$$\frac{dy}{dx} = 1 - y, y(10) = 2$$

- (c) Write sage commands to solve the following differential equation with the boundary conditions :

$$x_0 = 0, y(x_0) = 1, x_1 = \frac{\pi}{2}, y(x_1) = 4$$

$$\frac{d^2y}{dx^2} + y = 0. \qquad 2+(2+2)+4$$

Section - VI

Answer *any one* question.

12. (a) Write a program in sage to find the GCD of two numbers a, b using Euclidean Algorithm.
 (b) Write sage code for the following :
 Find the number of digits in $2021!$ and compute the number of zeros and the number of ones present in $2021!$. [Here $2021!$ means factorial of 2021 .]
 (c) Write your code in sage to find the root of the polynomial $f(x) = x^5 + x^4 + x^3 - x^2 + x - 1$ between -1 and 1 . 5+(1+1+1)+2

13. (a) Write a program in sage to find the greatest of three given integers a, b, c .
 (b) Write a program in sage to find the sum of the following series for any finite n :
 $2.3 + 4.5 + 6.7 + \dots + 2n(2n + 1)$.

- (c) Consider the following program :

```
i=0
while i < 5:
    print(i)
    i=i+1
    if i==3:
        break
    else:
        print(0)
```

What will be the output of the program segment?

4+3+3

Please Turn Over

[Scientific Computing with R]

1. Answer *all* questions with proper explanation / justification (*one* mark for correct answer and *one* mark for justification) : 2×10

(a) What should we type to get the value of $\tan(90^\circ)$ in R?

(i) $\tan 90^\circ$

(ii) $N\left(\tan\left(\frac{\pi}{2}\right)\right)$

(iii) $\tan(\pi/2)$

(iv) None of these.

(b) The value of factorial of 3^{91} in R is

(i) not given

(ii) $\text{factorial}(3^{91})$

(iii) Infinity

(iv) None of these.

(c) Which of the following R codes will print "Hello, world"?

(i)

```
> f <- function() {
+   cat("Hello, world!\n")
+ }
> f()
```

(ii)

```
> f <- function() {
+   cat("Hello, World!\n")
+ }
< f()
```

(iii)

```
> f <- function() {
+   cat("Hello world!\n")
+ }
>>= f()
```

(iv)

```
> f <- function() {
-   cat("Hello World!\n")
+ }
<= f()
```

(d) What will be the output of the following R code?

```
> f <- function (a, b) {
+   a^2
+ }
> f (2)
```

(i) 4

(ii) 3

(iii) 2

(iv) 5.

(e) What will be the output of the following R code snippet?

```
> paste ("a", "b", se = ":",)
```

(i) "a+b"

(ii) "a=b"

(iii) "ab:"

(iv) None of these.

(f) The output of the following R program will be :

```
> n <- 0
> square <- 0
> while (square <= 100) {
+ n <- n + 1
+ square <- n ^ 2
+ }
> print(n)
> print(square)
```

- (i) 11
100
- (ii) 11
121
- (iii) 11
100
- (iv) None of these.
- (g) What will be the output of the following R code?
options(digits = 16)
20/6
- (i) 3.33
- (ii) 3.333
- (iii) 3.3333333
- (iv) 3.3333333333333333.
- (h) Which one of the following is the correct command to obtain the following matrix?
- $$A = \begin{bmatrix} 20 & 25 & 30 \\ 20 & 15 & 10 \end{bmatrix}$$
- (i) `A <-matrix(nrow=2, ncol=3, data=c(20,20,25,15,30,10), byrow=T)`
- (ii) `A <-matrix(nrow=3, ncol=2, data=c(20,20,25,15,30,10), byrow=T)`
- (iii) `A <-matrix(nrow=2, ncol=3, data=c(20,25,30,20,15,10), byrow=F)`
- (iv) `A <-matrix(nrow=2, ncol=3, data=c(20,25,30,20,15,10), byrow=T)`
- (i) How does a vector differ from a list?
- (i) Vector and list are same and can be used interchangeably.
- (ii) Vector is used only for numeric data, while list is used for any kind of data.
- (iii) A vector contains item of a single data type, while a list can contain items of different data types.
- (iv) Vector is like array, while list is like data frame.
- (j) What is the output of the code given below?
A=10
B=20
print(A,B)
- (i) 10
- (ii) 20
- (iii) 10 20
- (iv) Error.

Section - I

Answer *any one* question.

2. Find the outputs of the following R commands :

- (a) `x=5;y=2; x=x^2+y^2; y=(x+1)/5; y`
- (b) `x=1:7; x=x+3; x`

Please Turn Over

(c) $x=8$; $x=(x+1)**2$; $x=\text{sqrt}(x)$; x

(d) $A=\text{matrix}(c(1,2,3,4), \text{nrow}=2)$; $B=\text{matrix}(c(4,3,2,1), \text{nrow}=2)$

$X=A*B+2$; X

2+2+2+4

3. Find the outputs of the following R commands.

(a) $x=7:2$; $x=x^2+1$; x

(b) $x=7:2$; $x=x^2+2$; $x=\text{length}(x)$; x

(c) $A=\text{matrix}(c(1,2,3,4), \text{nrow}=2)$; $B=\text{matrix}(c(-4,3,-2,1), \text{nrow}=2)$

$X=A\%*\%B+2$; X

3+3+4

Section - II

Answer *any one* question.

4. Write R code to plot the graph of the following in one plot :

(a) graph of the function $f(x) = x^2+1$ in range $(-2,2)$

(b) graph of the tangent line $f'(x)=2x$ in range $(-2,2)$ at the point $(1,2)$.

5+5

5. Write R code(s) for plotting

(a) the graph of polar curve

$$r = 1+2\cos(\theta), 0 \leq \theta \leq 2\pi$$

(b) the following functions in one plot :

(i) $f(x) = |x|+1$, in $-2 \leq x \leq 2$ with blue color

(ii) $g(x)=\sin(x) + 2 \cos(x)$, in $-2 \leq x \leq 2$ with red color.

4+3+3

Section - III

Answer *any one* question.

6. Write R code to do the following for the function $f(x) = x^3 + x$ in $[-2,2]$:

(a) find $f'(x)$

(b) find $f''(x)$

(c) plot $f'(x)$ and $f''(x)$ in $[-2,2]$ with blue and green colors respectively.

3+3+2+2

7. Write R commands to find the values of the following integrals :

(a) $\int_{-\infty}^{+\infty} e^{-x^2} dx$

(b) $\int_{-\pi}^{+\pi} \sin(5x) dx$

(9)

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(c) $\int_{-\pi/4}^{+\pi/4} \tan(x)dx$

(d) $\int_{-\pi/4}^{+\pi/4} e^x \cdot \tan(x)dx$

(e) $\int_{-1}^{+1} \frac{(1+x^3)}{(1+x^2)} dx.$

2+2+2+2

Section - IV

Answer *any one* question.

8. Without using inbuilt functions, write R code to find the mean and median of the data set $c(3,4,7,1,7,4,8,9,10,11,2,12)$. 5+5
9. Without using inbuilt functions, write R code to find GCD and LCM of the numbers 12 and 75. 6+4

Section - V

Answer *any one* question.

10. Given the matrix, $A = \begin{bmatrix} 2 & 5 & 3 \\ 4 & 5 & 2 \\ 6 & 3 & 4 \end{bmatrix}$, write R programs

- (a) to find A^3
- (b) to find the determinant and the inverse of A
- (c) to find the inverse of A as well as to check whether the result is correct. 3+4+3

11. (a) Solve the following system of equations by a R program :

$$4x + y - z = 3$$

$$2x + 7y + z = 19$$

$$x - 3y + 12z = 31.$$

- (b) Write a R program to find roots of the polynomial $x^7 + 3x^4 + 8x + 9$. 6+4

Section - VI

Answer *any one* question.

12. (a) Write a R program to print first n non-Fibonacci positive integers.
- (b) Write a R program to print all the prime numbers in the interval [131,291]. 5+5

Please Turn Over

13. (a) Write a R program to find the sum of the series :

$$1^2 + 2^2 + 3^2 + \dots + N^2.$$

(b) Discuss how we can solve the differential equation : $\frac{dx}{dt} = 1 - x$ using R commands. Also write the code for plotting the solution in a specific range. 5+5
