2021

PHYSICS — HONOURS

Paper : CC-1

(Mathematical Physics - I)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Evaluate $\lim\limits_{x\to\infty}(1+ax)e^{-bx};\ a,b>0$.

(b) Show that $(y+z)\,dx + x\,dy + x\,dz$ is an exact differential.

(c) Check whether the three vectors $(\hat{i}+\hat{j}), (\hat{i}-\hat{j})$ and $(\hat{j}-\hat{k})$ are linearly independent.

(d) Given $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\vec{F}=x^3y\,\hat{i}+y^2\,\hat{j}+x^2z\,\hat{k}$.

Calculate $(\vec{r}.\vec{\nabla})\vec{F}$.

(e) Show that the area bounded by a simple closed curve c lying in x-y plane is given by $\frac{1}{2}\oint_c (x\,dy - y\,dx)$.

(f) Given a unitary matrix U, show that $U^{-1}HU$ is Hermitian if H is a Hermitian matrix.

(g) Find a symmetric matrix S such that $Q=X^TSX$ where $Q=x_1^2+2x_1x_2-3x_2^2$ and $X=\begin{pmatrix}x_1\\x_2\end{pmatrix}$.

2. (a) Show that $\delta z = x\,dy - (y-x^2)\,dx$ is an inexact differential. Find a suitable integrating factor to make the equation exact. 1+3

(b) Find the particular integral of $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^x\sin x$. 4

(c) Find the coefficient of x^3 in the Taylor series expansion of $e^x\sin x$ about $x=0$. 2

Or,

(c) For what values of x the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ converges? **(For Syllabus : 2018-2019)** 2

3. (a) Using the concept of Wronskian, show that the functions 1, x and $\sin x$ are linearly independent.

(b) Show, using Lagrange's undetermined multipliers, that the axes of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ can be obtained by finding the maximum and minimum distance of a point on the ellipse to its centre.

(c) If $z=\sin\left(\frac{x}{y}\right)$, compute $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$.

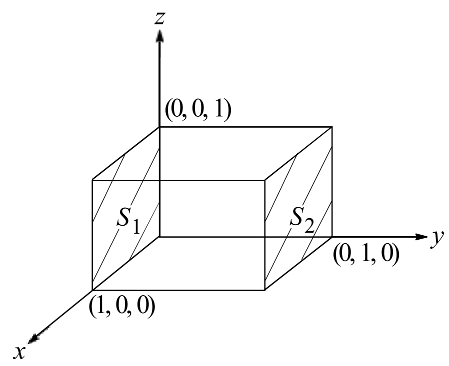
(d) Show that $(A_yB_x - A_xB_y)$ transforms as a scalar under rotation in the x-y plane, where $\vec{A}=A_x\hat{i}+A_y\hat{j}$ and $\vec{B}=B_x\hat{i}+B_y\hat{j}$. 2+3+2+3

4. (a) If the magnitude of a vector $\vec{A}(t)$ is constant with respect to time t, show that $\frac{d\vec{A}}{dt}$ is perpendicular to $\vec{A}$.

(b) Find the unit normal to the surface $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}=1$ at $\left(\sqrt{2}, 0, 2\sqrt{2}\right)$. Find the equation of tangent plane to the surface at the given point.

(c) Find the potential $\phi(x, y, z)$ for $\vec{F}=\left(3x^2yz+y+5\right)\hat{i}+\left(x^3z+x-z\right)\hat{j}+\left(x^3y-y+7\right)\hat{k}$, which has the value 10 at the origin. 1+(3+2)+4

5. (a) Compute $\iint_S \vec{F}.d\vec{s}$ over the surfaces S_1 and S_2, where $\vec{F}=x\hat{i}+y\hat{j}+z\hat{k}$.



(b) What do you mean by orthogonal curvilinear coordinate system? Find the unit vectors of the spherical polar coordinate system in terms of $\hat{i}, \hat{j}$ and $\hat{k}$.

(c) Use Gauss' theorem to convert the volume integral $\iiint_V \left(\phi\nabla^2\psi + \vec{\nabla}\phi.\vec{\nabla}\psi\right) dV$ to a surface integral over the boundary enclosing V. Here $\phi(x, y, z)$ and $\psi(x, y, z)$ are two scalar fields. 3+4+3

6. (a) Show that the eigenvalues λ of a two-dimensional invertible real-valued matrix A obeying $A^{-1} = A^{\dagger}$ satisfy $|\lambda|^2 = 1$.

(b) Show that if B is an invertible matrix, then $B^{-1}e^{A}B = e^{B^{-1}AB}$

(c) Solve the system of equations by Matrix method

$$\frac{dy}{dt} = z$$
$$\frac{dz}{dt} = -y$$

with initial conditions $y(0) = 1$ and $\dot{y}(0) = 0$. 3+3+4

7. (a) Let a unitary matrix U can be written as $U = A + iB$, where A and B are Hermitian matrices having non-degenerate eigenvalues. Show that $A^2 + B^2 = I$.

(b) Show that for a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\det A = \frac{1}{2}\left[(Tr\,A)^2 - Tr\left(A^2\right)\right]$, where Tr represents trace.

(c) If a matrix $A = \begin{pmatrix} a & h \\ h & b \end{pmatrix}$ is transformed to the diagonal form $B = UAU^{-1}$ where $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, show that $\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$. 3+3+4
