2021

PHYSICS — HONOURS

Paper : CC-2

(Mechanics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.* 1** and ***any four*** from the rest.

1. Answer ***any five*** questions : 2×5

(a) Show that Newton's second law of motion remains invariant under Galilean transformation.

(b) Show that for a body of variable mass m moving with momentum $\vec{p}$ and acted upon by an external force $\vec{F}$

$$\frac{d}{dt}(mT) = \vec{F} \cdot \vec{p}$$

where T is the kinetic energy of the body.

(c) A particle is acted upon by a force $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$. Show that the force is conservative. Find the corresponding scalar potential.

(d) An object made from a thin wire is shaped like a square with side of length L and total mass M. What is the moment of inertia of this object about an axis that passes through the centre of the square and is perpendicular to the plane of it?

(e) The trajectory of a particle of mass m is described in cylindrical polar coordinates by $\dot{z} = 0$, $\dot{\theta} = \omega$ and $r(t) = r_0 \sinh(\omega t)$; where ω and r_0 are constants. Compute the radial component of force.

(f) Prove that the motion of a particle under the influence of a central force is confined on a plane.

(g) State the equation of continuity.

***Or*, (Syllabus 2018-2019)**

What is geometrical moment of inertia?

2. (a) The force on a particle is given by $\vec{F} = cx\hat{j}$, where c is a positive constant. The particle moves in xy-plane in counterclockwise direction around a square loop whose corners are at (0, 0), (L, 0), (L, L) and (0, L). Calculate the work done on the particle by the force during one complete trip around the square. From the result, can you say whether the force is conservative or non-conservative?

Please Turn Over

(b) A particle of mass $\frac{2}{3}$kg is subjected to a potential energy function $V(x) = \left(3x^2 - 2x^3\right)J$, where $x \geqslant 0$ and expressed in metres.

(i) What is the maximum value of the potential energy?

(ii) If the particle is released at $x = \frac{4}{3}$, find its velocity when it reaches $x = \frac{3}{2}$.

(c) Show that the equation of motion of a rocket in the absence of any external force is given by :

$$M = \frac{d\vec{v}}{dt} = \vec{u}\frac{dM}{dt},$$

where M and $\vec{v}$ are the instantaneous mass and velocity of the rocket respectively and $\vec{u}$ is the velocity of the spent fuel relative to the rocket body. (3+1)+(2+2)+2

3. (a) The total energy of a particle moving in a central force field is the sum of its kinetic energy and "effective" potential energy— Explain. Show that energy of the particle is conserved.

(b) The trajectory of a particle of mass m moving under the influence of a central force in plane polar coordinates is given by $r = r_0 e^{\alpha\theta}$, where r_0 and α are positive constants. The angular momentum of the particle is L and its total energy is zero. Express the potential energy function $V(r)$ in terms of m, L and α.

(c) Show that the kinetic energy of a system of particles is sum of total kinetic energy of the system of particles with respect to centre of mass and kinetic energy of a particle of mass M (total mass of the system of particles), moving with the velocity same as the velocity of the centre of mass $\left(\vec{V}_{CM}\right)$. (2+1)+3+4

4. (a) Two particles of masses m_1 and m_2 are moving with relative velocity $\vec{v}$ and the velocity of their centre of mass is $\vec{V}_{CM}$. If M is the total mass and μ is the reduced mass of the system, then prove that total kinetic energy of the system is $T = \frac{1}{2}MV_{CM}^2 + \frac{1}{2}\mu v^2$.

(b) For a homogeneous cube of mass M and side l set up principal axes with origin at centre of the cube. Determine the principal moments of inertia and set up equation of ellipsoid of inertia. 4+(1+3+2)

5. (a) Prove the relation $\vec{v} = \vec{\omega} \times \vec{r}$, symbols having their usual meanings.

(b) A body is dropped from rest at a height of 400m above the surface of the earth at a latitude of 45°N. Find the magnitude and direction of the deflection due to Coriolis force when the body touches the earth.

(c) An inhomogeneous solid cylinder of radius R and length l is rotating about its axis aligned in the vertical direction Z. Measuring z from the bottom of the cylinder along the axis, the density is given by $\rho_z = \rho_0\left(1-\frac{z}{2l}\right)$, where ρ_0 is a constant. Calculate the moment of inertia of the cylinder.

2+3+5

6. (a) A rigid body is rotating about the x axis. Find—

(i) the angular momentum vector $\vec{L}$

(ii) the condition for $\vec{\omega}$ and $\vec{L}$ to be parallel

(iii) the kinetic energy of the body under condition (ii) stated above.

(b) Define centre of mass of a system of particles. Show that centre of mass is a fixed point.

(c) A particle acted upon by a central force describes an orbit given by $r = a(1 + \cos\theta)$, where a is a constant. Show that the force is attractive and varies as r^{-4}. (1+1+1)+3+4

7. (a) State Bernoulli's theorem. Differentiate between Newtonian and non-Newtonian fluids.

(b) Use the equation $\vec{F} = \vec{\nabla}p$ for a fluid at rest to establish Pascal's law.

(c) Water flows through a horizontal tapering tube of circular cross-section, the diameter of the entrance and exit ends being 10 cm and 7.5 cm respectively. Calculate the velocities of flow at the two ends. The pressure difference between the two ends is 10 cm of mercury.

***Or*, [Syllabus 2018-2019]**

A rectangular metal bar of length 80 cm, breadth 1 cm and depth 0.5 cm is supported horizontally on two pegs at its ends. The depression produced in the middle by a load of 200 gm is 2 mm. Calculate Young's modulus of the material of the bar. (2+2)+3+3