2021

PHYSICS — HONOURS

Paper : DSE-B-2

(Advanced Statistical Mechanics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any five*** questions : 2×5

(a) Consider a system with N particles and three energy states. The states are $E_1 = 0$, $E_2 = K_BT$ and $E_3 = 2K_BT$. The total energy of the system is 3000 K_BT. Find the number of particles.

(b) Classify the following particles according to Fermi or Bose statistics :

(i) ^{3}He, (ii) H_2 molecule, (iii) $^6Li^+$ion, (iv) $^7Li^+$ion

(c) A classical gas of molecules, each of mass m, is in thermal equilibrium at absolute temperature T. If v_x, v_y and v_z are the three cartesian components of the velocity of a molecule, then calculate $\langle (v_x - v_y)^2 \rangle$.

(d) The Hamiltonian of a system in 1–D is $H = ap_x^2 + bx^2 + cx$. Find the value of $\langle H \rangle$.

(e) Show that the density operator for a pure state is idempotent.

(f) A random walker takes a step of unit length in the positive direction with the probability $^2/_3$ and a step of unit length in the negative direction with probability $^1/_3$. Calculate the mean displacement of the walker after n-step.

(g) What do you mean by coarse graining in non-equilibrium statistical mechanics?

Group - B

Answer ***any three*** questions.

2. Using the fact that the Gibbs free energy $G(N, p, T)$ of a thermodynamic system is an extensive property of the system, show that $G = N\frac{\partial G}{\partial N}$. Hence show that $G = N\mu$, where μ represents the chemical potential of the system. 5

3. Calculate the value of $-\frac{\partial f(\epsilon)}{\partial \epsilon}$ at $\epsilon = \mu$, where $f(\epsilon)$ represents the FD distribution function. Show that at $T \to 0, -\frac{\partial f(\epsilon)}{\partial \epsilon}$ at $\epsilon = \mu$ behaves as a delta function. 2+3

4. Consider free electrons in silver (Ag) at 300 K. Test whether it is a classical system or not.

Given : Density of Ag = 10·5 gm/cc.

Atomic weight = 107·9 gm.

Boltzmann constant $k_B = 1{\cdot}381\times10^{-23}\ J/K$. 5

5. Consider N distinguishable and non-interacting particles. The single particle energy spectrum is $\epsilon_n = n\epsilon$, with $n = 0, 1, 2,\ldots,\infty$ and degeneracy $g_n = n + 1$ ($\epsilon > 0$ is a constant). Compute the canonical partition function and the average energy. 5

6. A system has two spin states $S = +1$ or $S = -1$. The system contains N number of molecules. If there is interaction among the particles write down the Hamiltonian using Ising model. Now using Bragg Williums approximation show that the Hamiltonian can be expressed as

$$-\frac{1}{2}\gamma\,\epsilon\, Nm^2 - \mu BmN. \qquad 5$$

where m is the long range order parameter, γ is the number of nearest neighbour of the particle, ϵ is the interaction energy and μ is the magnetic moment of the particle.

Group - C

Answer ***any four*** questions.

7. A system has two energy levels with energy 0 and ϵ. The system may be either unoccupied or occupied by a single particle in any one of its energy level. Calculate the grand canonical partition function of the system. Determine the average occupancy $\langle N\rangle$ of the system. Find an expression for thermal average energy of the system. 4+3+3

8. (a) Consider a spin $\frac{1}{2}$ system with a pure state

$$|\alpha\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(i) find corresponding density matrix

(ii) find the density matrix for 'up' spin state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

(b) The Hamiltonian of an electron in a magnetic field $\vec{B}$ is given by $H = -\mu_B\vec{\sigma}\cdot\vec{B}$, where $\vec{\sigma}$ is Pauli spin operator and μ_B stands for the Bohr magneton. Construct the density matrix in the diagonalized representation of σ_z. Also calculate the $\langle\sigma_z\rangle$. the z-axis being taken along the field direction. (2+2)+3+3

9. (a) Show that the pressure of the weakly degenerate bosons is less than that of the classical gas. [Given the relation

$$pV = Nk_B T \frac{g_{5/2}(z)}{g_{3/2}(z)}$$

where

$$g_\nu(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^\nu} = z + \frac{z^2}{2^\nu} + \frac{z^3}{3^\nu} + \ldots \Bigg]$$

(b) Experimentally Bose–Einstein condensation was produced in a vapour of ^{87}Rb atoms at a number density of $2{\cdot}5 \times 10^{12}$ cm^{-3}. Calculate the critical temperature.

(c) Explain why Bose condensation is not possible in one dimension. 3+3+4

10. (a) Consider a gas of free electrons confined inside a three-dimensional box. The z-component of the magnetic moment of each electron is $\pm\,\mu_B$. In the presence of a magnetic field B pointing in the z-direction, each 'up' state acquires an additional energy of $-\,\mu_B B$, while each 'down' state acquires an additional energy of $+\,\mu_B B$.

(i) Explain why you would expect the magnetization of a degenerate electron gas to be substantially less than that of the paramagnets for a given number of particles at a given field strength.

(ii) Obtain an expression for the net magnetization of this system at $T = 0$.

(b) Consider a degenerate electron gas in which essentially all of the electrons are highly relativistic ($\epsilon >> mc^2$), so that their energies are $\epsilon = pc$. Calculate the chemical potential of such a system. (2+5)+3

11. (a) Calculate the fluctuations $\langle N^2 \rangle - \langle N \rangle^2$ in number density N in grand cannonical ensemble. Hence evaluate the relative root mean square fluctuation in N.

(b) Obtain the partition function of an ideal gas in grand cannonical ensemble. Hence calculate the internal energy and grand potential of the system. (4+1)+(2+2+1)

12. (a) Estimate the Fermi energy and Fermi temperature of an white dwarf star. Given : mass of proton $9{\cdot}1 \times 10^{24}$ gm, mass density of star $\sim 10^7$ g/cc.

(b) Explain why the electron gas in white dwarf star is highly degenerate.

(c) Consider a person walking randomly in 1D from a point. Find the probability that the person after N displacement will be at a distance $x = ml$, where m is an integer and l is the step size. 10
