2021

STATISTICS — HONOURS — PRACTICAL

Seventh Paper

(Group - A)

Full Marks : 50

The figures in the margin indicate full marks.

(Notations have their usual significance)

1. The observations to follow, taken on 10 incoming shipments of chemicals in drums arriving at a warehouse, show number of drums in shipment (X_1), total weight of shipment (X_2, in hundred pounds) and number of minutes required to handle the shipment (Y).

i	1	2	3	4	5	6	7	8	9	10
X_1	7	18	5	14	11	5	23	9	16	5
X_2	5·11	16·72	3·20	7·03	10·98	4·04	22·07	7·03	10·62	4·76
Y	58	152	41	93	101	38	203	78	117	44

Assume that the multiple regression of Y on X_1 and X_2 with independent normal error terms is appropriate.

(a) Obtain the estimated regression function. How are b_1 and b_2 here interpreted?

(b) Test whether β_1 and β_2 are significant or not.

(c) Test whether there is a regression relation, using level of significance of 0.05.

(d) Calculate the coefficient of determination R^2 and interpret it.

(e) Also calculate $r^2_{Y1.2}$ and $r^2_{Y2.1}$. Interpret the result. 4+4+2+2+4

2. Let X equal the number of pounds of butterfat produced by a Holstein cow during the 305-day milking period following the birth of a calf. Assume that the distribution of X is $N(\mu, 140^2)$. To test the null hypothesis $H_0 : \mu = 715$ against the alternative hypothesis $H_1 : \mu < 715$, let the critical region be defined by $C = \{\bar{x} : \bar{x} \leq 668.94\}, \bar{x}$ is the sample mean of $n = 25$ butterfat weights from 25 cows selected at random.

(a) Define the power function $K(\mu)$ for this test.

(b) What is the significance level of this test?

(c) What are the values of K(668.94) and K(622.88)?

(d) Sketch a graph of the power function. 2+2+2+3

3. We wish to compare comprehensive strengths of concrete corresponding to $a = 3$ different drying methods (treatments). Concrete is mixed in batches that are just large enough to produce three cylinders. Although care is taken to achieve uniformity, we expect some variability among the $b = 5$ batches used to obtain the following comprehensive strengths.

Treatment	Batch				
	B1	B2	B3	B4	B5
A1	52	47	44	51	42
A2	60	55	49	52	43
A3	56	48	45	44	38

(a) Use 5% significance level and test $H_A : \alpha_1 = \alpha_2 = \alpha_3 = 0$.

(b) Use 5% significance level and test $H_B : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$. 6

4. For developing countries in Africa and the Americas, let p_1 and p_2 be the respective proportions of babies with a low birth weight (below 2500 grams). We shall test $H_0 : p_1 = p_2$ against the alternative $H_0 : p_1 > p_2$. If a random sample of size $n_1 = 900$ taken from Africa yielded $y_1 = 135$ babies with a low birth weight and a random sample of size $n_2 = 700$ taken from Americas yielded $y_2 = 77$ babies with a low birth weight, what is your conclusion? 4

5. Let X and Y equal the number of miligrams of tar in filtered and non-filtered cigarettes respectively. Assume that the distributions of X and Y are $N(\mu_X, \sigma^2)$ and $N(\mu_Y, \sigma^2)$, repectively. We shall test the null hypothesis $H_0 : \mu_X - \mu_Y = 0$ against the alternative hypothesis $H_1 : \mu_X - \mu_Y < 0$ using random samples of sizes $n = 9$ and $m = 11$ observations of X and Y, respectively.

Observations of X : 0·9, 1·1, 0·1, 0·7, 0·4, 0·9, 0·8, 1·0, 0·4

Observations of Y : 1·5, 0·9, 1·6, 0·5, 1·4, 1·9, 1·0, 1·2, 1·3, 1·6, 2·1

Calculate the value of the test statistic and conclude your result. Also obtain a 95% confidence interval for the difference in means of X and Y. 5+3

6. Let X be N(μ, 100).

(a) To test $H_0 : \mu = 230$ against $H_1 : \mu > 230$, what is the critical region specified by a likelihood ratio test criterion based on a random sample of size n from the above population?

(b) Is this test uniformly most powerful?

(c) If a random sample of size $n = 16$ yielded $\bar{x}$ (sample mean) = 232.6, is H_0 rejected at a significance level $\alpha = 0.1$?

(d) What is the p-value of this test? 2+2+2+1