2021

STATISTICS — HONOURS — PRACTICAL

Paper : CC-2P

Full Marks : 30

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words
as far as practicable.

1. A bank considers changing its credit card policy. Currently 5% of credit card owners are not able to pay their bills in any month, i.e., they never pay their bills. Among those who are generally able to pay their bills, there is still a 20% probability that the bill is paid too late in a particular month.

    (a) What is the probability that someone is not paying his bill in time in a particular month?

    (b) A credit card owner did not pay his bill in time in a particular month. What is the probability that he never pays back the money? Should the bank consider blocking the credit card if a customer does not pay his bill on time? 3+(2+1)

2. A space shuttle has six O-rings and that each O-ring fails with probability

$$p(t) = \frac{e^{a+bt}}{1 + e^{a+bt}},$$

where $a = 5.085$, $b = -0.1156$, and t is the temperature (in degrees Fahrenheit) at the time of launch of the space shuttle.

    (a) Let X be the number of failing O-rings at launch temperature 31ºF. Find the probability mass function of X.

    (b) What is the probability that at least one O-ring fails?

    (c) Suppose it is decided that a space shuttle will be launched at 81ºF. What is the probability that during 12 launches no O-ring will fail, but that at least one O-ring will fail during the 13th launch of a space shuttle? 3+1+2

3. Suppose a class has twelve students, with roll numbers 1, 2,...,12. A teacher randomly notes the roll numbers of five of these twelve students. Let X be the maximum of these five roll numbers. Find the expected value and the variance of X. 7

4. Suppose an airline accepted 104 reservations for a flight with 100 seats. They know that 95 reservations went to regular commuters who will show up for sure. The other 9 passengers will show up with a 60% chance, independently of each other.

    (a) Find the probability that the flight will be overbooked, i.e., more passengers will show up than seats are available.

    (b) Find the probability that there will be empty seats.

    (c) Let X be the number of passengers turned away. Find expectation and variance of X. 3+3+5