# 2021

## STATISTICS — HONOURS

### Paper : CC-2

### Full Marks : 50

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**1.** Answer ***any five*** questions : 2×5

(a) An urn has two blue and three red marbles. Three marbles are drawn one by one, without replacement, and their colour noted. Define the sample space for this experiment.

(b) Suppose that $A, B$ and $C$ are mutually independent events and that $P(A) = 0.5$, $P(B) = 0.8$ and $P(C) = 0.9$. Find the probability that exactly two of the three events occur.

(c) Suppose $P(A) = P(B) = 0.9$. Suggest a useful lower bound on $P(B \mid A)$.

(d) A fair six-faced dice is rolled twice, and let $X$ be the larger of the two outcomes if they are different and the common value if they are same. Find the probability mass function of $X$.

(e) Give an example of a probability density function $f$ which has discontinuities at $x = 0$, 1 and 2.

(f) Give an example of a cumulative distribution function which is not left continuous.

(g) An urn contains eight cards, four marked $A$ and four marked $B$. Randomly select cards, one at a time, without replacement. After four cards have been selected, let $X$ equal the difference between the number of $A$ and $B$ cards that have been selected. What is the mode of $X$?

(h) Consider a non-negative random variable $X$ with $E(X) = 1$. Stating the necessary result suggest an upper bound for $P(X > 1.5)$.

**2.** Answer ***any two*** questions : 5×2

(a) A doctor assumes that a patient has one of three diseases $d_1$, $d_2$ or $d_3$. Before any test, he assumes an equal probability for each disease. He carries out a test that will be positive with probability 0.8 if the patient has $d_1$, 0.6 if he has disease $d_2$ and 0.4 if he has disease $d_3$. Given that the outcome of the test was positive, what probabilities should the doctor now assign to the three possible diseases?

(b) Distinguish between mutual independence and pairwise independence of a set of events. Suppose a fair coin is tossed twice. In connection with this experiment construct three events which are pairwise independent but not mutually independent.

(c) Define cumulative distribution function of a random variable and state all its properties. Give an example of a function which, except for any one, satisfies all the other properties of a cumulative distribution function.

**Please Turn Over**

**3.** Answer ***any three*** questions : 10×3

(a) What are the limitations of the 'classical definition' of probability? Illustrate, using a suitable example of your choice, how one can overcome at least one of these limitations with the help of the axiomatic definition.

(b) Find an expression for the probability of occurrence of at least two out of four events $A_1, A_2, A_3, A_4$ which are not necessarily mutually exclusive. Illustrate the expression obtained using a suitable example of your choice.

(c) A restaurant critic goes to a place twice. If she has an unsatisfactory experience during both visits, she will go once more. Otherwise she will not make any more visits. Assume that the results for different visits are independent and that the probability of a satisfactory experience in any one visit is 0.8.

(i) Find the probability of at least two unsatisfactory visits.

(ii) Find the conditional probability of at least one satisfactory visit given at least one unsatisfactory visit.

(iii) Find the mean and variance of the number of visits paid by the critic.

(d) Consider a random variable $X$ with probability density function

$$f(x) = k\theta x e^{-\theta x^2}, \quad 0 < x < \infty, \; k, \theta > 0.$$

Given that the median of $X$ is 1, find the values of $k$ and $\theta$. Give example of another continuous random variable $Y$ having the same support as that of $X$ and median equal to 1. Compare the means of $X$ and $Y$, provided they exist.

(e) Define expectation, variance and $q$th quantile $(0 < q < 1)$ of a continuous random variable $X$. If $\xi_q$ be the $q$th quantile $(0 < q < 1)$ of a continuous random variable $X$ for which $E(X) = \mu$ and $\text{Var}(X) = \sigma^2 \; (< \infty)$, show that $\mu - \sigma\sqrt{\frac{1-q}{q}} \le \xi_q \le \mu + \sigma\sqrt{\frac{q}{1-q}}$.

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