2021

STATISTICS — HONOURS

Paper : CC-4

(Probability and Probability Distributions - II)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any five*** questions : 2×5

(a) Define probability generating function. State one use of it.

(b) Suggest a uniform distribution with variance 12. What is the mean of your suggested uniform distribution?

(c) Write down the probability mass function of a Poisson random variable with coefficient of variation 4.

(d) Suppose $X \sim$ binomial ($2n$, 0.5). Find the value of $\lim_{n\to\infty} P(X=n)$.

(e) Give a real-life example of hypergeometric distribution.

(f) State two properties of the distribution function of a two-dimensional random vector.

(g) State two properties of lognormal distribution.

(h) Write down the probability density function of a bivariate normal random vector where the means are same, the variances are same and the correlation coefficient is $-\frac{1}{7}$.

2. Answer ***any two*** questions : 5×2

(a) Give one example each of a discrete and a continuous random variable X for which

$$P(X \geq s+t \mid X \geq t) = P(X \geq s) \ \forall\ s,t.$$

Justify your answers.

(b) Show that the expected number of independent tosses of a coin we need to perform until we get a head is equal to the reciprocal of the probability that any toss results in a head. Give an intuitive justification of the result.

(c) Write a short note on Pareto distribution.

3. Answer ***any three*** questions : 10×3

(a) Find the variance of a random variable X with moment generating function

$$M(t) = e^{2(e^t - 1)}, -\infty < t < \infty .$$

Suggest a moment generating function of a random variable Y such that $\dfrac{\text{Variance of } X}{\text{Variance of } Y} = 2$.

Justify your answer.

(b) Suppose X is a normal random variable with mean α and variance β^2. Find the moment generating function of X. Find a random variable Z, a function of X, such that $E(Z) = e^{2(\beta^2 - \alpha)}$.

(c) Define independence of two discrete random variables. Give an example of two dependent discrete random variables. For two independent discrete random variables X and Y, what can you say about $E(XY) - E(X)E(Y)$? Justify your answer.

(d) Suppose there are 13 different types of balls and suppose that each time one obtains a ball, it is equally likely to be any one of the above 13 types. Find the expected number of different types that are contained in a set of 13 balls.

(e) Write a note on trinomial distribution.