2022

ECONOMICS — HONOURS

Paper : CC-4

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any ten*** questions : 2×10

(a) Consider the function $f(x_1, x_2) = x_1x_2 + x_2^2$. Find the corresponding marginal functions and comment on their degree of homogeneity.

(b) For the total cost function $C = y^2 + 10y + 25$; $y > 0$, show that when average cost (AC) curve is horizontal, then AC = MC (Marginal Cost).

(c) Find the point elasticity of demand (w.r.t. own price) for the demand function $x = 100p^{-2}$.

(d) Find the extreme values of the function $y = 0.5x^3 - 3x^2 + 6x + 10$ and determine whether it gives a maxima or a minima.

(e) Find the marginal product functions for the Cobb-Dauglas production function : $y = 10x_1^{1/2}x_2^{1/2}$.

(f) For the funciton $f(x_1, x_2) = x_1^2x_2$, verify the Young's Theorem.

(g) Determine the MRS for the utility function $u(x_1, x_2) = ax_1 + bx_2$.

(h) Show that the quadratic equation formed by the following matrix product is positive definite.

$$[x_1\ x_2]\begin{bmatrix}4 & 2\\ 2 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

(i) State the duality theorem in the context of linear programming problems.

(j) Find the inflexion point for the function $y = \ln x + {}^1/_x$.

(k) A production function is given by : $Q(L) = 12L^2 - \frac{1}{20}L^3$; where L denotes the number of workers. What size of workforce maximises output per worker?

(l) For the function $x = 5.e^t$, show that the relative rate of increase $\frac{\dot{x}}{x}$ is constant.

(m) Mohan lives in two periods, today and tomorrow. At the beginning of each period he earns ₹ 5,000. If the interest rate in each period is 0.25, find the present value of her lifetime income.

(n) Write down the Kuhn-Tucker conditions for the following optimization problem :

Maximize $z = 2x_1 - x_1^2 + x_2$

Subject to $2x_1 + 3x_2 \leq 6$

$2x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

(o) Discuss the nature of the following time paths :

(i) $y_t = 5\left(-\frac{1}{10}\right)^t + 3$

(ii) $y_t = 2\left(\frac{1}{3}\right)^t$

Group - B

Answer ***any three*** questions.

2. Comment on the quasiconcavity/quasiconvexity of the following function :

$y = 2x_1^{1/2} x_2^{1/2};\ x_1, x_2 > 0$. 5

3. Given $C = 102 + 0.7y$, $I = 150 - 100r$; $M_S = 300$; $M_T = 0.25y$, $M_P = 124 - 200r$ where C = consumption, Y = income, I = investment, r = rate of interest, M_S = Money supply, M_T = Transaction demand for money. M_P = Speculative demand for money. Find (i) The equilibrium level of income and the rate of interest. (ii) The levels of C, I, M_T and M_P at equilibrium. 3+2

4. What is a level curve? Compute the slope of the level curves for the function $f(x_1, x_2) = 2x_1 + 3x_2$. 2+3

5. Examine whether the following functions are homothetic?

(a) $e^{x^2 y}$

(b) $2\log x + 3\log y$ 2½+2½

6. A consumer's utility function is given by $U = x^{\alpha} y^{\beta}$. Show that the absolute value of price and income elasticities for either good is equal to unity. 5

Group - C

Answer ***any three*** questions.

7. (a) Derive the indirect utility function in case of a Cobb-Dauglas utility function $u(x, y) = x^{\alpha} y^{\beta}$. Where $\alpha + \beta = 1$ and the budget equation is given by : $I = P_x \cdot x + P_y \cdot y$.

(b) Derive the compensated demand function for the utility function $u^{\circ} = q_1 q_2$ and the expenditure function $E = p_1 q_1 + p_2 q_2$. Verify the Shephard's Lemma. 4+(3+3)

8. (a) (i) Suppose that y is a function of x_1 and x_2 given by :

$y = -(x_1 - 1)^2 - (x_2 - 2)^2 + 10$

where y represents an individual's health (measured on a scale of 0 to 10), and x_1 and x_2 might be daily dosage of two health enhancing drugs. The objective is to maximise y. But the choice of x_1 and x_2 is constrained by the fact that an individual can tolerate only one drug does per day i.e $x_1 + x_2 = 1$. Find out the optimal combination of drug that will maximise the health standard subject to the constraint.

(ii) What would have been the optimal choice had there been no constraint. How does the maximum value of y changes in this unconstrained case, compared to the constrained one.

(b) Consider the following profit equation of a firm producing two products x and y :

$$\pi = 80x - 1.5x^2 - xy - y^2 + 60y$$

Find the profit maximizing combination of output and the level of maximum profit of the firm.

(2+3)+5

9. (a) Let the demand and supply function for a commodity be :

$Q_d = D(P, t_0)$; $\frac{\partial D}{\partial P} < 0, \frac{\partial D}{\partial t_0} > 0$ *and* $Q_S = Q_{S_0}$. Where t_0 is consumer's taste for the commodity and where both partial derivatives are continuous.

(i) Write the equilibrium condition as a single equation.

(ii) Is the implicit function applicable?

(iii) How would the equilibrium price vary with consumer taste.

(b) A firm uses capital K, labour L and land T to produce Q units of output, where $Q = K^{2/3} + L^{1/2} + T^{1/3}$. Suppose that the firm is paid a positive price p for each unit it produces and the positive prices it pays per unit of capital, labour and land are r, w and q respectively.

(i) Find the values of K, L and T that maximises firm's profit.

(ii) Show that $\frac{\partial Q^*}{\partial r} = -\frac{\partial K^*}{\partial p}$,, where Q^* denotes the optimal level of output and K^* denotes the optimal level of capital stock.

(1+2+2)+(3+2)

10. (a) Consider the following market model :

$$Q_t^d = Q_t^s \qquad Q_t^d = 20 - 3P_t$$

$$Q_t^s = \begin{cases} -10+3P_t^* & \text{for } t = 1,2,\ldots \\ 0 & \text{for } t = 0 \end{cases}$$

where P_t^* is the expected price at t-th period given that

$$P_t^* = P_{t-1}^* + \alpha\left[P_{t-1} - P_{t-1}^*\right]_{t=2,3,\ldots}^{0<\alpha<1} \quad P_t^* = P_0; t = 1$$

Find the time path of price.

Please Turn Over

(b) Consider the linear difference equation of the Cob-Web model :

$$P_{t+1} = \frac{a+\gamma}{\beta} - \frac{\partial}{\beta} \cdot P_t \left(\frac{\partial}{\beta} > 0 \right)$$

Draw a phase line to ascertain the nature of the time path. 5+5

11. Consider the following linear programming problem :

Maximize $x_1 + x_2$

Subject to $x_1 + 2x_2 \leq 14$

$2x_1 + x_2 \leq 13$

$x_1 \geq 0,\ x_2 \geq 0$

(a) Solve the problem graphically.

(b) Write down the dual of this problem.

(c) Use complementary slackness conditions to solve the dual.

(d) Check whether duality theorem holds. (3+2+3+2)

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