2022

MATHEMATICS — HONOURS

Paper : DSE-A(2)-3

(Fluid Statics and Elementary Fluid Dynamics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Symbols have their usual meanings.]

1. Answer ***all*** questions with proper explanation/justification (***one*** mark for correct answer and ***one*** mark for justification) : 2×10

(a) If a fluid is in equilibrium, then the pressure at a point is

(i) same at any temperature (ii) different in different direction

(iii) same in every direction (iv) none of these.

(b) The equation of free surface of an ocean is of the form

(i) $x^2 + y^2 + z^2 =$ constant

(ii) $x + y + z =$ constant

(iii) $x + y + z =$ constant, $x^2 + y^2 + z^2 =$ constant

(iv) $x^2 + y^2 =$ constant, $z =$ constant.

(c) If p_1 and p_2 are the pressures at the points of depth h_1 and h_2 respectively in a homogeneous fluid, then

(i) $p_1 \propto h_1$ and $p_2 \propto h_2$ (ii) $p_1 + p_2 \propto h_1 + h_2$

(iii) $p_1 - p_2 \propto h_1 - h_2$ (iv) none of these.

(d) Effect of viscosity is neglected in

(i) Real fluid (ii) Newtonian fluid

(iii) Ideal fluid (iv) Non-Newtonian fluid.

(e) Isothermal process is characterized by

(i) $\frac{p}{\rho} =$ constant (ii) $pT =$ constant

(iii) $pv^\gamma =$ constant (iv) $p\rho^\gamma =$ constant.

(f) The centre of gravity of the displaced homogeneous fluid is

(i) centre of pressure (ii) centre of buoyancy

(iii) metacentre (iv) centre of force.

(g) An incompressible steady flow pattern (u, v, w) is given by $u = x^3 + 2z^2$, $w = y^3 - 2yz$. Which one of the following could be a form of $v(x, y, z)$ so that continuity equation is satisfied?

(i) $x^3 + y^2$ (ii) $-3x^2y + 2yz$

(iii) $-3x^2y + y^2$ (iv) $-x^3 + 2yz$.

(h) Density is considered to be constant in

(i) Inviscid fluid (ii) Real fluid

(iii) Newtonian fluid (iv) Incompressible fluid.

(i) The equation of streamline is

(i) $\vec{q} \times \overrightarrow{dr} = \vec{0}$ (ii) $\vec{q} . \overrightarrow{dr} = 0$

(iii) $\vec{r} . \overrightarrow{dq} = 0$ (iv) None of the above.

(j) Given steady, incompressible velocity distribution $\vec{V} = 3x\hat{i} + Cy\hat{j} + 4z\hat{k}$, where C is a constant. If conservation of mass is satisfied, the value of C should be

(i) -3 (ii) 0

(iii) -7 (iv) -4.

Unit – 1

2. Answer ***any one*** question :

(a) (i) Define body and surface forces on a fluid element.

(ii) Prove that in a fluid at rest, the surfaces of equi-pressure cut the lines of force at right angles. 2+3

(b) A solid triangular prism, the faces of which include angles α, β, γ is placed in any position entirely within a liquid. If P, Q, R be the thrusts on the three faces respectively opposite to the angles α, β, γ, prove that $P \operatorname{cosec} \alpha + Q \operatorname{cosec} \beta + R \operatorname{cosec} \gamma$ is invariable so long as the depth of the centre of gravity of the prism is unchanged. 5

Unit – 2

3. Answer ***any two*** questions :

(a) (i) A liquid of volume V is at rest under the force $X=-\frac{\mu x}{a^2}, Y=-\frac{\mu y}{b^2}, Z=-\frac{\mu z}{c^2}$. Find the pressure at any point of the liquid and the surface of equal pressure.

(ii) Determine the C.P. of a vertical circular area immersed in a liquid with its centre at a depth h below the free surface. 5+5

(b) (i) One end of a horizontal pipe of circular section closed by a vertical door hinged to the pipe at the top. Show that the moment about the hinge of the liquid pressure is $\frac{5}{4}\pi\rho g a^4$, when it is full of liquid of density ρ, 'a' being the radius of the section and g the acceleration due to gravity.

(ii) A solid hemisphere is placed with its base inclined to the surface of a liquid, in which it is just totally immersed, at a given angle α, in such a way that the resultant thrust on the portion of the surface is equal to twice the weight of the liquid displaced. Prove that $\tan\alpha = 2$. 5+5

(c) (i) Prove that the tangent plane at any point on the surface of buoyancy is parallel to the corresponding position of the plane of floatation.

(ii) A solid cylinder of radius a and length h is floating with its axis vertical. Show that the equilibrium will be stable if $\frac{a^2}{h'} > 2(h-h')$, where h' is the length of the axis immersed. 5+5

(d) (i) Derive the expressions for pressure and density in an isothermal atmosphere at a height z above the sea level, considering gravity to be constant.

(ii) If the law connecting the pressure and density of the air is $p = k\rho^n$, prove that neglecting variations of gravity and temperature, the height of the atmosphere would be $\frac{n}{n-1}$ times the height of the homogeneous atmosphere, k being a constant. (3+3)+4

Unit – 3

4. Answer ***any one*** question :

(a) (i) Distinguish uniform and non-uniform flows .

(ii) A velocity field is given by $\vec{q} = x^3\hat{i} + xy^3\hat{j}$. Find the equation of streamlines of the flow.

(iii) Describe the Lagrangian and Eulerian methods of describing the fluid flow. 2+3+5

(b) (i) The velocity components of inviscid, incompressible, steady flow with negligible body force in spherical polar co-ordinates are given by $u_r = V\left(1-\frac{R^3}{r^3}\right)\cos\theta,\ u_\theta = -V\left(1+\frac{R^3}{2r^3}\right)\sin\theta,\ u_\phi = 0,$ where R and V are constants. Prove that it is a solution of momentum equation of motion.

(ii) A velocity field is given by $\vec{V} = 4tx\hat{i} - 2t^2 y\hat{j} + 4xz\hat{k}$. Is this flow steady? Compute acceleration vector at the point $(x, y, z) = (-1, 1, 0)$. 5+(2+3)

Unit – 4

5. Answer ***any two*** questions : 5×2

(a) What is conservation of momentum and hence write the momentum equation of fluid.

(b) If the lines of motion are curves on the surfaces of cones having their vertices at the origin and the axis of z for common surface, prove that the equation of continuity is $\frac{\partial\rho}{\partial r} + \frac{\partial(\rho u)}{\partial r} + \frac{2\rho u}{r} + \frac{\operatorname{cosec}\theta}{r}\frac{\partial(\rho w)}{\partial\theta} = 0$, where u and w are the velocity components in the directions in which r and ϕ increase.

(c) Find the values of l, m, n for which the velocity profile $q = \frac{x+lr}{r(x+r)}\hat{i} + \frac{y+mr}{r(x+r)}\hat{j} + \frac{z+nr}{r(x+r)}\hat{k}$ satisfies the equation of continuity for a liquid.

(d) The velocity distribution for flow in a long circular tube of radius R is given by the one-dimensional expression $\vec{V} = u\hat{i} = u_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]\hat{i}$. For this profile obtain expression for the volume flow rate through a section normal to the axis of the tube.