2022

PHYSICS — HONOURS

(Syllabus : 2019-2020 & 2018-2019)

Paper : CC-1

[Mathematical Physics - I]

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Evaluate $\underset{x\to 4}{\mathrm{Lt}}\ \dfrac{\sqrt{x}-2}{x-4}$, if it exists.

(b) Plot schematically xe^{-x} vs. x for $0 \le x < \infty$.

(c) Find whether vectors $2\hat{i}+5\hat{j}+3\hat{k}, \hat{i}+\hat{j}+\hat{k}$ and $4\hat{i}-2\hat{j}$ are linearly independent.

(d) Find the Taylor series expansion of $\ln x$ about $x = 2$.

(e) Determine Wronskian of the two solutions to the following differential equation $x^4y''-2x^3y'-x^8y=0$.

(f) Find the eigenvalues of the matrix $\begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix}$.

(g) Prove that the eigenvalue of a skew Hermitian matrix is purely imaginary.

2. (a) Plot the function $f(x) = x^2$ and its first derivative.

(b) Find the constant term in the expansion of $\left(x^2+\dfrac{1}{x^2}\right)^6$.

(c) Check whether $df = (3x^2-3ay)dx + (3y^2-3ax)dy$ is an exact differential.

(d) Find the series expansion of $\dfrac{1}{1-x}$. Mention its interval of convergence. 2+2+2+(2+2)

Or,

(For Syllabus : 2018-2019)

(d) Consider

$$f(x)=\begin{cases} x^m \sin\left(\dfrac{1}{x^n}\right) & x \neq 0 \\ 0 & x=0 \end{cases}$$

Find out the condition on m and n so that $f(x)$ is differentiable at $x=0$. 4

3. (a) Solve the equation :

$$e^x \sin y\, dx + (e^x+1)\cos y\, dy = 0$$

(b) Check whether the function $\sin x$, e^x and e^{-x} are linearly independent or not.

(c) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of the box that requires minimum surface area for its construction. 3+3+4

4. (a) Find the directional derivative of $\phi = x^2y + xz$ at (1,2,–1) in the direction of $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$.

(b) If $\vec{\nabla}\times\vec{F} = 4x\hat{i} - 2x\hat{j} + cz\hat{k}$ then find c.

(c) Consider two vector fields $\vec{F}_1 = 2x\,\hat{i} - 2yz\,\hat{j} - y^2\hat{k}$ and $\vec{F}_2 = y\hat{i} - x\hat{j}$. Which of the above is a conservative field? For the non-conservative field, calculate the work done if it acts on an object moving from (–1, –1) to (1, 1) along the straight line joining the two points. 3+2+(2+3)

5. (a) A fluid motion is given by $\vec{v} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$. Show that the motion is irrotational and hence find the velocity potential.

(b) Use Green's theorem to evaluate

$$\oint_c [(xy+y^2)dx + x^2dy]$$

where c is the triangle with vertices (0, 0), (1, 0) and (1, 2).

(c) Evaluate $\iint \vec{A}.d\vec{s}$ where $\vec{A} = x\cos^2 y\,\hat{i} + xz\,\hat{j} + z\sin^2 y\,\hat{k}$ over the surface of a sphere with centre at the origin and of radius 3 unit. 3+4+3

6. (a) The matrices A and B satisfy $(AB)^T + B^{-1}A = 0$. Prove that if B is orthogonal, then A is anti- symmetric.

(b) Find out the eigenvalues and normalized eigenvectors of the matrix $M = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} (a \neq 0)$. Find out M^n where 'n' is a positive integer.

(c) Explain whether the inverse of the following matrix exists $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix}$. 2+(1+2+2)+3

7. (a) If $A^2 = A$, then show that $e^{\theta A} = \mathbb{I} + (e^{\theta} - 1)A$.

(b) Given $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$; what can you comment on the nature of eigenvalues of A without solving the characteristic equation.

(c) Show that two similar matrices have the same characteristic polynomial.

(d) Show that for a orthogonal matrix, each column is orthogonal to other ones. 2+2+3+3