2022

PHYSICS — HONOURS

Paper : CC-5

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Explain what is the physical meaning of the constant term in a Fourier series expansion.

(b) Using the Generating function for the Legendre polynomials

$$(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)\, t^n,$$

show that $p_n(-1) = (-1)^n$, where n is a positive integer.

(c) Discuss about the singularities of the following equations :

(i) $\frac{d^2y}{dx^2} - \frac{6}{x^2}y = 0$.

(ii) $\frac{d^2y}{dx^2} - \frac{6}{x^3}y = 0$.

(d) In an α-particle counting experiment, the number of α-particles is recorded in each minute for two hours. The total number of particles is 500. In how many 1-minute intervals do you expect no particle?

Or, [Syllabus 2018-19]

Which symmetry of the Lagrangian does the conservation of Hamiltonian (Energy) comes from? Justify.

(e) Obtain the Parseval identity for a Fourier series.

Or, [Syllabus 2018-19]

Show that if one adds, to the Lagrangian of a system, a total time derivative of a function of co-ordinate and time only, the equation of motion remains invariant.

(f) Show that $\Gamma(z+1) = z\,\Gamma(z)$ for any z.

(g) Show that $B(p, q) = B(q, p)$, where $B(q, p)$ is the beta function.

2. (a) Find the Fourier transform $g(k)$ of the function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-x^2/2\sigma^2\right]$$

and plot both $f(x)$ and $g(k)$.

(b) What is the physical meaning of σ? Find the corresponding quantity of $g(k)$ and show how they are related.

(c) When $\sigma \to 0$ limit is taken what will happen to $f(x)$ and to $g(k)$? 5+3+2

Or, [Syllabus 2018-19]

(a) Find the shortest distance between two nearby points in 2-dimensional Euclidian Space using variational principle.

(b) Two bodies of mass m_1 and m_2 are hanging under gravity from the two ends of an inextensible string of length l which goes over a frictionless, massless pulley. Is it a constrained system? Find the Lagrangian and equations of motion of the masses. What is the force of constraint here? 4+(1+2+2+1)

3. (a) Evaluate $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$ using Beta and Gamma functions.

(b) Show that $B(n, n) = B(n, \tfrac{1}{2})/2^{2n-1}$

(c) Show that $\Gamma(2n) = \frac{1}{\sqrt{\pi}} 2^{2n-1} \Gamma(n)\Gamma\left(n + \tfrac{1}{2}\right)$

(d) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ 2+3+2+3

4. (a) Find the Fourier Series for the periodic function $f(x) = e^x$, $-\pi \le x < \pi$, with a period 2π.

(b) If $f(x) = f(-x)$ and $g(x) = -g(-x)$, show which terms should be present in the Fourier expansion of $f(x)$ and $g(x)$, with period $-1 \le x < 1$.

(c) Find the Fourier series expansion for $f(x) = \cos^2 x$. 5+3+2

5. (a) Prove that the Legendre polynomials of different orders are orthogonal.

(b) Using the expression for Bessel's function

$$J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\,\Gamma(m+r+1)} \left(\frac{x}{2}\right)^{n+2r},$$

Show that,

(i) $J_{m+1}(x) + J_{m-1}(x) = \frac{2m}{x} J_m(x)$

(ii) $J_{m-1}(x) - J_{m+1}(x) = 2J'_m(x)$ 4+(3+3)

6. (a) How does the Fourier transform $g(k)$ of a function $f(x)$ change under the translation $x \rightarrow x + a$, where a is some constant?

(b) If $g(k)$ is the Fourier transform of $f(x)$, what is the Fourier transform of $f^*(x)$?

(c) Show that in a certain limit Binomial distribution can be converted to Gaussian (Normal) distribution.

(d) Show that Fourier transform of $f(x) = 1$ has the properties of Dirac's δ-function by integrating from $-L$ to $+L$ and then taking $L \rightarrow \infty$ limit. 2+2+3+3

Or, [Syllabus 2018-19]

(a) Show that if the Lagrangian is invariant under rigid rotation, then angular momentum of the system is conserved.

(b) Let us consider the Lagrangian in polar coordinate $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\,\dot{\theta}^2\right) - V(r)$.

Find the equations of motions. Is there any cyclic coordinate? What are the conserved quantities and why? 5+(2+1+2)

7. (a) A bar 10 cm long with insulated sides is initially at 100°C. Starting at $t = 0$, its ends are held at 0°C. Find the temperature distribution in the bar at time t. Use the Heat flow equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}$$

where $u(x,t)$ is the temperature, α is a constant.

(b) Consider the vibration of a circular membrane obeying

$$\nabla^2\Psi = \frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2}$$

where $\Psi = \Psi(r, \theta, t)$.

Find the solution of the equation by separation of variables method. What will be the boundary conditions needed here? Take the radius of the membrane to be R. The circumference is held fixed. 4+6