2022

PHYSICS — HONOURS

(2019–2020 Syllabus)

Paper : CC-10

(Quantum Mechanics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Find the degeneracy of an energy level with principal quantum number n for a hydrogen atom.

(b) Can Lithium ($Z = 3$) give rise to Normal Zeeman effect? Explain.

(c) Show that $[L^2, L_{\pm}] = 0$ when $L_{\pm} = L_x \pm L_y$. The symbols have their usual meanings.

(d) Find the eigenvalues and eigenfunctions of angular momentum operator $\hat{L}_z = -i\hbar \frac{\partial}{\partial \phi}$.

(e) Define degeneracy. What is meant by degree of degeneracy?

(f) What are L-S and J-J coupling?

(g) What is Hund's rule?

2. (a) A particle is described by the following wave function (at $t = 0$) :

$$\psi(x,0) = \sqrt{\frac{1}{6}}\psi_1(x) + \frac{i}{\sqrt{2}}\psi_2(x) + \frac{1}{\sqrt{3}}\psi_3(x)$$

ψ_1, ψ_2, ψ_3 are the first three energy eigenstates of the harmonic oscillator. How will such state evolve in time? Find the average energy of the particle.

(b) Obtain the expectation values of $\hat{x}$, $\hat{p}$ and x^2 for the ground state of simple harmonic oscillator. (2+2)+(2+2+2)

3. For a linear harmonic operator with $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$, $\hat{a} = \frac{1}{\sqrt{2m\hbar\omega}}[\hat{p} - im\omega\hat{x}]$

and $a^{\dagger} = \frac{1}{\sqrt{2m\hbar\omega}}[\hat{p} + im\omega\hat{x}]$

(a) Find $[\hat{H}, \hat{a}], [\hat{H}, \hat{a}^\dagger], [\hat{H}, \hat{a}\hat{a}^\dagger]$

(b) Show that $\hat{H} = \left(\hat{a}\hat{a}^\dagger - \frac{1}{2}\right)h\omega$

(c) Show that $\hat{H}\,\hat{a}\psi_n(x) = \left(n - \frac{1}{2}\right)h\omega\psi_n(x)$

If energy corresponding to a particular state $\psi_n(x)$ is $\left(n + \frac{1}{2}\right)h\omega$. 4+3+3

4. The initial $(t = 0)$ wave function of a free particle is described by the Gaussian wave packet $\psi_0(x) = Ae^{-ax^2}$, where A and $\alpha(\alpha > 0)$ are constants.

(a) Normalize the wave function $\psi_0(x)$.

(b) Find the wave function $\psi(x, t)$ of the free particle at a later time t.

(c) Calculate the probability density and hence show that the wave packet of the free particle broadens spatially with time.

(d) Plot the probability density at time $t = 0$ and time $t > 0$ with x. 2+4+3+1

5. (a) Express the operators L^2 and L_z in spherical polar coordinate system. Hence verify that $\psi(\theta, \phi) = \left(\frac{15}{32\pi}\right)^{\frac{1}{2}} \sin^2\theta e^{2i\phi}$ is an eigenfunction of both L^2 and L_z. Find the eigenvalues.

(b) A spin $\frac{1}{2}$ particle is in a state $|\alpha\rangle = \frac{1}{\sqrt{30}}\begin{pmatrix} 2-i \\ 5 \end{pmatrix}$. If S_z is measured, then find the probabilities of getting eigenvalues $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$. (4+2+1)+3

6. (a) Consider two non-interacting identical particles of mass m in an infinite square well. The single particle state is given as

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right),$$

when $E_n = n^2K$, a is the width of the well from $x = 0$ to $x = a$ and $K = \frac{\pi^2\hbar^2}{2ma^2}$.

(i) Find the ground state normalized wave functions and energies if the particles are identical bosons and identical fermions.

(ii) Find the normalized wave function for first excited state and the corresponding energy if the particles are identical bosons.

(b) (i) In Stern-Gerlach experiment, why is it necessary to use beam of neutral atoms and not of ions?

(ii) In this experiment, a beam of neutral nickel atom splits into nine components. What is the angular momentum of a nickel atom in its ground state? (4+2)+(2+2)

7. (a) Using first-order perturbation theory, calculate the shift of energy with respect to the Bohr energy of hydrogen atom due to spin-orbit interaction term.

(b) Hence, derive the complete fine-structure formula for hydrogen atom if the relativistic correction to kinetic energy is $E_r^{\,l} = -\dfrac{(E_n)^2}{2mc^2}\left(\dfrac{4n}{l+\dfrac{1}{2}} - 3\right)$, where symbols have their usual meanings.

(c) Write down the electronic structure of Carbon Atom. Hence, express the ground state of the atom with the help of Term symbol. 4+3+3

(Syllabus : 2018-2019)

[Analog Systems and Applications]

Full Marks : 50

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Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

   (a) What do you mean by dynamic resistance of a diode? How is it related with junction temperature of the diode?

   (b) What are the fundamental differences between class-A and class-C amplifiers?

   (c) The inverting input terminal of an inverting amplifier is virtually grounded. Explain.

   (d) What is an emitter follower?

   (e) Which type of carrier is responsible for reverse saturation current in a diode?

   (f) Compare ideal and practical characteristics of OPAMP.

   (g) Explain Barkhausen's criterion for sustained oscillation.

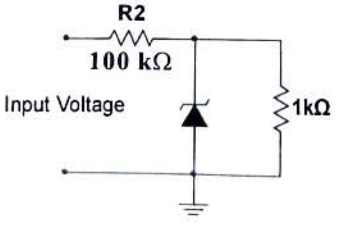
2. (a) When a silicon diode is reverse biased at 300 K, the current through the diode is 1 μA. Find the current through the diode for a forward bias of 0.6V.

   Given Boltzmann constant,

   $$k = 1.38\times10^{-23}\ \text{m}^2\text{kg s}^{-2}\text{K}^{-1}.$$

   (b) In a half-wave rectifier, prove that the efficiency of rectification is about 20.3% when the rectifier delivers maximum DC power to the load.

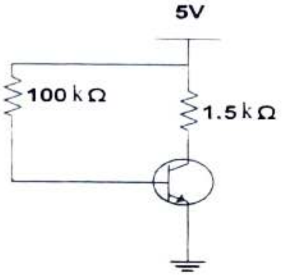
   (c) A 5V Zener diode is connected in series with the resistance of 100 Ω and a load resistance of 1 kΩ is connected across Zener diode. The minimum Zener current is zero and the maximum Zener current should not exceed 20 mA. Calculate the operating range of the input voltage. 2+4+4



3. (a) Explain the current amplification factors α and β for CB and CE configuration respectively. Obtain the relation between them.

   (b) The current gain of a transistor CE in is 135. Calculate the emitter current if the collector current configuration is 49.3 mA.

(c) In the circuit shown, the transistor has $\beta = 100$. Assume, $V_{BE} = 0.6V$ at active region, $V_{BE(sat)} = 0.8V$ and $V_{CE(sat)} = 0.2V$ at saturation. Find in which region the transistor is operating. (2+2)+3+3



4. (a) Define h_{ie} and h_{fe} of a common emitter circuit.

(b) Draw the common source drain characteristics of a JFET and explain the behaviour in different regions.

(c) Explain briefly the basic principle of a photodiode. 2+(2+3)+3

5. (a) Draw the frequency response curve of a single stage CE-amplifier. Indicate the upper and lower cut-off frequencies on it.

(b) Explain with circuit diagram, the operation of a series regulated power supply mentioning the role of each component of the circuit.

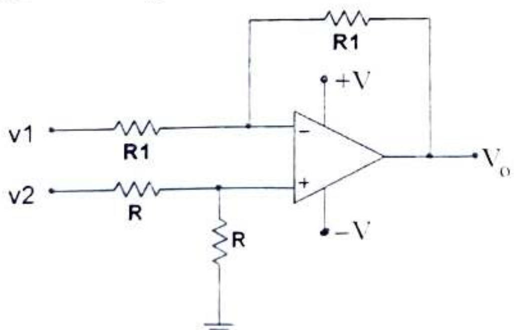
(c) Show that the bandwidth of an amplifier can be increased with the help of negative feedback but the gain bandwidth product remains unchanged. (1+1)+4+4

6. (a) The voltage gain of a transistor amplifier is 60. The input and output resistances are 1kΩ and 50kΩ respectively. The amplifier is provided with 10% negative voltage feedback in series with the input. Calculate the voltage gain, input resistance and the output resistance of the amplifier.

(b) A two state RC coupled amplifier uses transistors having $h_{ie} = 1k\Omega$, $h_{fe} = 100$. The load resistance is 2.2 kΩ. Calculate the overall mid-frequency gain. Neglect the effect of source resistance and biasing resistances.

(c) Draw the circuit diagram of a phase shift oscillator. Write down the expression for frequency of the oscillator. What is the minimum value of gain of the amplifier for sustained oscillation? 4+3+3

7. (a) What is the output of the following circuit? What happens if positions of R and C be interchanged?



(b) Draw the circuit of a Schmitt trigger using OPAMP and explain its operation.

(c) Find the output of the following circuit. 3+4+3