2022

PHYSICS — HONOURS

(Syllabus : 2018-2019 and 2019-2020)

Paper : CC-8

(Mathematical Physics III)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Prove that $\lim\limits_{z \to 0}\left(\dfrac{z}{\bar{z}}\right)$ does not exist, where $\bar{z}$ is the complex conjugate of z.

(b) Expand $\dfrac{1}{z-2}$ in Taylor's series for $|z| < 2$, where z is complex.

(c) Find the residue of $f(z) = \dfrac{\sinh z}{z^5}$ at its singularity at $z = 0$.

(d) Write the Lagrangian of a simple pendulum and obtain its equation of motion using the Lagrangian.

Or, **(for 2018-2019 Syllabus only)**

Find the Fourier transform of $f(t) = e^{-i\omega t}$.

(e) Show using Variational principle that two Lagrangians whose difference is a total time derivative of a function of coordinates give same equation of motion.

Or, **(for 2018-2019 Syllabus only)**

For a random variable X, $\langle X \rangle = 2$ and $\langle X^2 \rangle = 8$. Calculate the standard deviation.

(f) Show that the length of a moving rod is invariant in two inertial reference frames according to Galilean transformations.

(g) The mean lifetime of a muon at rest is 2.2 μs. Calculate the average distance that it will travel in vacuum before decay, if it starts moving with velocity 0.9c.

2. (a) (i) Prove that $u = 2x(1 - y)$ is harmonic.

(ii) Find a function v such that $f(z) = u + iv$ is analytic.

(iii) Express $f(z)$ in terms of z.

(b) Prove that the real and imaginary parts of an analytic function of a complex variable when expressed in polar form satisfy the Laplace's equation in polar form given by

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = 0.$$

(c) Locate and name all the singularities of

$$f(z) = \frac{z^8 + z^4 + 2}{(z-1)^3 (3z+2)^2}$$

(1+2+2)+3+2

3. (a) Find the Laurent series expansion of the function $f(z) = \dfrac{1}{z^2(1-z)}$ in the domain $0 < |z| < 1$.

(b) Evaluate the following integrals :

(i) $\displaystyle\oint_C \frac{e^{2z}}{(z+1)^3}\,dz \quad C: |z| = 2$

(ii) $\displaystyle\int_0^\infty \frac{\cos ax}{x^2+1} \quad a < 0$.

3+(3+4)

4. (a) Space-time coordinates of a pair of events in an inertial frame S are $A\left(\frac{a}{c}, a, 0, 0\right)$ and $B\left(\frac{a}{2c}, 2a, 0, 0\right)$. Find the separation of the two events in an inertial frame S' in which two events are simultaneous. Also find the speed of S' with respect to S. Use the metric $(1, -1, -1, -1)$.

(b) An inertial frame S' is moving at a speed $c/_2$ away from another inertial frame S along common $x - x'$ axis. As observed from S', a particle is moving with speed $c/_2$ in y' direction. Find the speed of the particle as seen from S.

(c) Consider three inertial frames S, S' and S''. S' is moving with respect to S along the common x–x' axis with a velocity v. S'' is moving with respect to S′ along common x'–x'' axis with velocity v'. If the velocity of S'' with respect to S is v'', then show that $\gamma'' = \gamma\gamma'\left(1+\beta\beta'\right)$ where $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$ and similarly for γ' and γ'', also, $\beta = \dfrac{v}{c}$ and $\beta^1 = \dfrac{v^1}{c}$

(2+1)+4+3

5. (a) Particle of mass M initially at rest, decays into two pieces each of mass m. Find out the speed of each particle.

(b) A particle of mass m whose total energy is twice its rest mass energy collides with an identical particle at rest. After collision, they stick together. Find the mass of the resulting composite particle. What is its velocity?

(c) Show that under Lorentz transformation $p^{\mu}p_{\mu}$ is invariant where p^{μ} is the 4-momentum of the particle. Use $(1, -1, -1, -1)$ metric. 3+(2+2)+3

6. (a) Using Variational principle find the Curve of shortest distance between two points in a plane.

(b) Consider the Lagrangian

$$L = \frac{1}{2}MV^2 + q\,\vec{V}.\vec{A},$$

where q is a constant scalar and $\vec{A}$ is a constant vector.

(i) Find the generalized momentum.

(ii) Find the Hamiltonian.

(c) A particle of mass m is constrained to move on a vertical circular loop of radius R. The gravity acts downwards. Construct the Lagrangian of the system. 3+(2+2)+3

***Or,* (for 2018-2019 Syllabus only)**

(a) Find the probability density function $f(x)$ for the position of a particle executing SHM on $(-a, a)$ along the x-axis.

(b) Let X be a random variable having a normalized density function

$$f(x) = \begin{cases} Ce^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find :

(i) the value of $|C|$ where C is a constant.

(ii) the variance σ^2.

(iii) the probability $P(1 \leq x \leq 2)$. 3+(2+3+2)

7. A particle of mass m is thrown with a velocity u making an angle α with the x-axis. The gravity acts downwards. Consider the motion in X-Y plane.

(a) Construct the Lagrangian.

(b) Find the equation of motion.

(c) Find if there is any cyclic coordinate.

(d) Construct the Hamiltonian.

(e) Using Hamilton's equation of motion, Show that $\frac{dH}{dt} = 0$. 2+2+1+2+3

Or, (for 2018-2019 Syllabus only)

8. (a) Find the Fourier sine transform of e^{-x}, $x \geq 0$.

(b) Using the results in (a), show that

$$\int_0^{\infty} \frac{x \sin mx}{x^2+1} dx = \frac{\pi}{2} e^{-m}, \ m > 0.$$

(c) Find the Fourier transform of $f(x) = Ae^{-x^2/2\sigma^2}$. 4+3+3