2022

PHYSICS — HONOURS

Paper : CC-12

(Syllabus : 2019-2020)

[Statistical Physics]

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.* 1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) A linear harmonic oscillator has energy in the interval E to $E + \Delta E$. By means of a suitable diagram, demarcate the region of phase space where the oscillator has all its possible microstates.

(b) State the postulate of equal a priori probability.

(c) What is λ-transition in liquid Helium?

(d) Sketch the spectral distribution graphs (u_λ vs. λ) of black-body radiation for two different temperatures T_1 and T_2 ($T_2 > T_1$). Do they intersect at any value of λ?

(e) Show that for a canonical system, mean energy

$$\langle E \rangle = -\frac{\partial}{\partial \beta}(\ln Z),$$

where Z is the canonical partition function.

(f) Show that the single particle occupation probability for a Fermion at T K with an energy interval $\pm kT$ around Fermi energy has an approximate range of 0.46.

(g) The concentration of electron in Cu is 8.5×10^{28} m^{-3}. Calculate whether the electron gas in Cu is degenerate or non-degenerate at $T = 500$ K.

(h) In how many ways can five Bosons be arranged in three quantum states?

2. (a) From the consideration of statistical mechanics under what condition will system 'A' (with volume 'V'_A, energy 'E'_A, number of particles 'N'_A and number of microstates Ω_A) only capable of exchanging energy with another system 'B' (with volume 'V'_B, energy 'E'_B, number of particles 'N'_B and number of microstates Ω_B) achieve equilibrium when the composite system consisting of the two systems is an isolated system?

(b) If in addition to energy the two systems can perform work on each other, what would be the condition of equilibrium?

(c) Assuming the entropy S is a function of the number of microstates Ω, show that the additive property of S and the multiplicative property of Ω necessarily require $S \propto \ln \Omega$. 4+2+4

Please Turn Over

3. Consider a classical ideal gas consisting of N identical and indistinguishable molecules of mass m enclosed in a container of volume V.

   (a) Calculate single particle partition function (ξ) and hence write down the partition function (Z) for the system by taking into account the fact that gas molecules are indistinguishable.

   (b) Calculate the mean energy of the system.

   (c) Calculate Helmholtz free energy of the system and hence derive Sackur Tetrode equation.

   (3+1)+2+(2+2)

4. (a) A particle in 1D has energy

$$E = \frac{p^2}{2m} + \lambda q^4,$$

   where q and p denote the generalized coordinate and momentum, respectively. Show that heat capacity of a gas comprising of N such particles is

$$C_V = \frac{3}{4}Nk.$$

   (b) The grand canonical partition function for an ideal gas is given by

$$Z_G\,(T, V, \mu) = \exp\left[e^{\mu/kT}\,\frac{V}{\lambda^3}\right],$$

   where $\lambda = \dfrac{h}{\sqrt{2\pi mkT}}$

   is the thermal de Broglie wavelength. Calculate average number of particles and hence calculate the equation of state of the system.

   (c) Why microcanonical, canonical and grand canonical ensembles give almost same results for a system with large number of particles? 4+3+3

5. (a) From Bose-Einstein distribution function, obtain Planck's law of black-body radiation. Hence derive Wein distribution law and Rayleigh-Jeans law.

   (b) What do you mean by ultraviolet catastrophe?

   (c) Assuming Sun radiates maximum energy at λ = 6000Å, calculate the approximate wavelength at which human body radiates maximum energy. Temperature of Sun is 6000°C. (4+2)+2+2

   (symbols have their usual significance)

6. (a) Consider two indistinguishable particles (1 and 2) which may exist in two different states ('a' and 'b'). The wave function for the entire system will be one of the following two possible forms :

$$\Psi_1 = \frac{1}{\sqrt{2}}\left[\psi_a(1)\,\psi_b(2) + \psi_a(2)\,\psi_b(1)\right] \text{ and } \Psi_2 = \frac{1}{\sqrt{2}}\left[\psi_a(1)\,\psi_b(2) - \psi_a(2)\,\psi_b(1)\right].$$

   Identify with reason the wave function that will represent the system when (i) both are Bosons, (ii) both are Fermions.

(b) Consider a system of two identical fermions which may occupy any of the three energy states 0, ε and 2ε. The system is in thermal equilibrium at absolute temperature T. Determine the partition function and the average energy of the system.

(c) Which statistics (Bose-Einstein or Fermi-Dirac) will apply to—

(i) alpha particles

(ii) Helium-3 atoms

(iii) Deuterium nuclei

(iv) Neutrons? 4+4+2

7. (a) Calculate the average energy of a strongly degenerate Fermi gas at $T = 0\text{K}$.

(b) The specific heat of silver at low temperatures can be represented by $C_V = \gamma T + \alpha T^3$ where γ and α are constants. Explain the origin of the term linear in T.

(c) In terms of number density (n) and absolute temperature (T), find the condition to be satisfied by an ideal gas for the onset of quantum effects. 5+3+2

(Syllabus : 2018-2019)

[Solid State Physics]

Full Marks : 50

Answer ***question no.*** **1** and ***any four*** questions from the rest.

2×5

1. Answer ***any five*** questions :

(a) Calculate the density of atoms in (111) planes of fcc aluminium whose lattice parameter is 4.05Å.

(b) Find out the packing fraction of a simple cubic structure.

(c) Is Meissner effect consistent with the disappearance of resistivity in a superconductor? Explain.

(d) With the help of M-H plot distinguish between type-I and type-II superconductors.

(e) Will the Hall coefficient change sign if one reverses the direction of the applied magnetic field?— Explain.

(f) Sketch the spontaneous magnetization of a ferromagnet as a function of temperature. Indicate the universal feature associated with the graph.

(g) The Fermi level for potassium is 2.1 eV at T = 0K. What is the classical and quantum average energy of the electrons at T = 0K?

2. (a) Write down the Laue equations and arrive at the Bragg's Law therefrom.

(b) What is a reciprocal lattice and how do you construct it?

(c) A beam of thermal neutrons emitted from the opening of the reactor is diffracted by the (111) planes of nickel crystal at an angle of 28°30′. Calculate the effective temperature of the neutrons. Nickel has fcc structure and its lattice parameter 3.52Å. 3+(2+1)+4

3. (a) Starting from the equation

$$\frac{P\sin\alpha a}{\alpha a}+\cos\alpha a=\cos Ka$$

where $P=\frac{mV_0ab}{\hbar^2}$, $\alpha^2=\frac{2mE}{\hbar^2}$ and symbols have their usual meaning. Find the lowest energy band using Kronig-Penney model for $P \ll 1$.

(b) Schematically represent the variation of velocity, effective man, and acceleration of an electron moving in a periodic potential.

(c) The Hall coefficient of a specimen is -7.35×10^{-5} m^3c^{-1}. What is the nature of the semiconductor? If the conductivity is 200 $m^{-1}\Omega^{-1}$, calculate the density and mobility of the charge carrier.

4+3+(1+2)

4. (a) The phonon dispersion relation for a vibrating diatomic chain in which alternate atoms are M_1 and M_2 is given by

$$\omega^2 = K_1\left(\frac{1}{M_1}+\frac{1}{M_2}\right) \pm K_1\left[\left(\frac{1}{M_1}+\frac{1}{M_2}\right)^2 - \frac{4\sin^2 Ka}{M_1 M_2}\right]^{\frac{1}{2}}$$

(K_1 = force constant; $K = \frac{2\pi}{\lambda}$)

Identify and obtain the minimum and maximum angular frequencies of the acoustical and optical branch.

(b) How does Debye model differ from the Einstein model of lattice heat capacity?

(c) Visible light of wavelength 5000 Å undergoes scattering from a crystal of refractive index 1.5. Calculate the maximum frequency of the phonon generated. (2+2)+2+4

5. (a) A paramagnetic atom having permanent moment $\vec{\mu}$ with a given resultant quantum number $\vec{J}$ is placed in a uniform magnetic field $\vec{B}$. Obtain an expression of the magnetisation as a function of $\vec{B}$ and temperature T. Hence obtain Curie's Law in the appropriate limit.

(b) Explain the hysteresis phenomena on the basis of domain theory.

(c) Atomic weight and density of iron are 55.847 and 7.87×10^3 kg m^{-3} respectively. If iron has a magnetic moment of 2.2 Bohr magneton, determine its spontaneous magnetization. (4+1)+3+2

6. (a) What do you mean by polarization of a solid?

(b) Clearly explain the basic assumptions and derive the Clausius-Mossotti relation for a dielectric. Explain how it is modified when more than one dielectric is present.

(c) The polarizability of NH_3 molecule in the gaseous state from the measurement of dielectric constant is found to be 2.42×10^{-39} Fm^2 at 309K and 1.74×10^{-39} Fm^2 at 448K. Compute the polarizability due to permanent dipole moment of NH_3 at 448K. 1+(4+2)+3

7. (a) What are Cooper pairs? How are they formed inside the superconducting material? What can you say about its spin state? How can you destroy the Cooper pairs?

(b) What is the importance of penetration depth in superconductivity? The penetration depths for lead are 396Å and 1730 Å at 3K and 7.1K respectively. Calculate the critical temperature for lead. (1+3+1+1)+(1+3)