2022

STATISTICS — HONOURS

Paper : CC-4

(Probability and Probability Distributions-II)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any five*** questions : 2×5

(a) Let X be a discrete random variable with probability generating function $P_X(t) = \frac{t}{9}(2t+1)(t+2)$. Find the distribution of X.

(b) Suppose X has a Geometric distribution with probability mass function

$$f(x) = p(1-p)^x,\ x = 0, 1, 2, \ldots;\ 0 < p < 1.$$

Given $\text{Var}(X) = 20$, find the value of p.

(c) Suppose X has a Poisson distribution with $P(X=0) = \frac{1}{e^{5/2}}$. What is the mode of X?

(d) Suppose X is a continuous random variable having Uniform distribution with mean 1 and variance $\frac{4}{3}$. Find $P(X > 0)$.

(e) Suppose that Z is a Standard Normal variable. Find $E(e^{|Z|})$.

(f) Suppose that X_1 and X_2 have the joint probability mass function

$$f(x_1, x_2) = p^2(1-p)^{x_2}, x_1 = 0,1,2,\ldots,x_2 \text{ and } x_2 = 0,1,2,\ldots$$

with $0 < p < 1$. Find the marginal distribution of X_1.

(g) Suppose X_1 and X_2 have the joint probability density function given by

$$f(x_1, x_2) = \begin{cases} 1, & \text{if } 0 < x_1, x_2 < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find $P(X_1 X_2 > a)$ for any $0 < a < 1$.

(h) Suppose (X_1, X_2) have a Trinomial distribution with parameters (n, p_1, p_2). Write down the conditional distribution of X_1 given $X_2 = x_2$.

2. Answer ***any two*** questions : 5×2

(a) Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with finite mean and common probability generating function P_X. Let N be a random variable with finite mean and independent of the X_i's with probability generating function P_N. Let $T_N = X_1 + X_2 + \ldots + X_N$. Find the probability generating function of T_N and hence find the mean of T_N.

(b) Find the mean deviation about mean of a Logistic distribution with parameters μ and σ.

(c) Let (X, Y) have joint density $f(x, y) = 2, 0 \le x \le y \le 1$. Find the joint cumulative distribution function of (X, Y). Also find the marginal densities of X and Y.

3. Answer ***any three*** questions : 10×3

(a) Find the probability mass function of a random variable X whose probability generating function is inversely proportional to $(3 - t^2)$. Also find the moment generating function of X. Hence obtain the mean and variance of X.

(b) For Negative Binomial distribution with parameters r and p, establish the recursive property of central moments. Hence find a measure of skewness and kurtosis of the Negative Binomial distribution and comment.

(c) Let X be a $N(\mu, \sigma^2)$ random variable truncated between a and b, where $a < b$. Find the moment generating function of X. Hence find the mean and variance of X.

(d) Give an example of a joint probability density function such that the marginal distribution of one of the two random variables is Exponential with mean 1 and the two random variables have a non-zero correlation coefficient. Find the marginal distribution of the other random variable. Also find the correlation coefficient between the two random variables.

(e) (i) Give an example of a bivariate distribution whose marginal distributions are Normal but the joint distribution is not Bivariate Normal.

(ii) Suppose X_1 and X_2 are independently distributed according to $N(0, 2\sigma^2)$ and $N(1, \sigma^2)$, respectively. Let $Y_1 = 2X_1 + X_2$ and $Y_2 = X_1 - 2X_2$. Find the moment generating function of (Y_1, Y_2) and hence comment on the distribution of (Y_1, Y_2).

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