2022

STATISTICS — HONOURS

Paper : SEC-B-1

(Monte Carlo Methods)

Full Marks : 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***any fifteen*** from question nos. **1-20**. 2×15

1. Define Pseudo random numbers.

2. Define inverse transform method for random variate generation.

3. Let $X_1, ..., X_n$ be iid r.v's distributed as $F_x(x)$.
Define $Y_{\mathrm{n}} = \max(X_1, ..., X_n)$ and $Y_1 = \min(X_1, ..., X_n)$.
How to generate Y_n and Y_1?

4. Let $X \sim$ Gamma (α, β) with integer α. How to generate a random variate from Gamma (α, β)?

5. State Box-Muller method for random number generation from normal distribution.

6. How to generate a random variate from Cauchy (α, β)?

7. Describe a method to approximate the value of π.

8. Consider the following random number generator
$$x_{n+1} = (6x_n + 5) \bmod 13$$
with $x_0 = 2$. After how many numbers, the seed x_0 will appear again?

9. Prove that if X is a r.v. with absolutely continuous distribution function P_X, the r.v $P_X(x)$ has uniform (0, 1) distribution.

10. How to generate a random sample of 1000 Bernoulli variates with success rate 30%?

11. If $\int_{-1}^{1} x^2 dx = E\left(g(X) \mid X \sim N(0,1)\right)$, determine the function g.

12. How will you approximate the integral $\int_0^{\infty} \exp(-x)dx$ by Monte Carlo method?

13. Describe an algorithm for generating a random observation from $P_x = \frac{1}{(b-a+1)}$, $x = a, a+1, ...b$ where b and a are integers and $b > a$.

14. Given 8 iid observations from $N(0, 1)$, how will you generate two observations from F-distribution with d.f (3, 5)?

15. Describe an algorithm for generating random observations from

$$f_X(x) = \frac{B^{-1}}{2} \exp\left[\frac{|x|}{B}\right],\ B > 0, -\infty < x < d.$$

16. Find an approximate value of $I = \int_0^1 e^{-x^3} dx$, by Monte Carlo integration.

17. Define Linear Congruential Generators. Give an example.

18. Let $X_1, ..., X_n$ be iid r.v's from exp(λ). Let $Y_1 = \min(X_1, ..., X_n)$. Describe an algorithm for generating Y_1.

19. If you are given uniform (0, 1) random numbers 0.67 and 0.11, how will you generate two observations from a Bernoulli (0.23) distribution? Are these observations independent?

20. Describe a method of generating a single observation from Bin(3, $^1/_5$) distribution.

Answer ***any six*** from question nos. **21-28**. 5×6

21. Describe an algorithm for generating from logistic distribution with the following pdf :

$$f_X(x) = \frac{\exp\left[-(x-\alpha)/_B\right]}{B\left(1+\exp\left[\left(-(x-\alpha)/_B\right)^2\right]\right)}; -\alpha < x < d, B > 0, \alpha \neq 0.$$

22. How will you generate a random permutation of items indexed 1, 2, 3, ..., N ?

23. Consider the problem of generating a simple random sample of size n from a population of size $N(N = 10^6)$. If you have a random number generator with period 10^9, what is the largest sample size you can draw in accordance with the difinition of SRS? Discuss.

24. Derive two different algorithms to generate a deal of hands for the game of bridge. (i.e., form 4 random disjoint sets each containing 13 of the integers 1, 2, ..., 52). Base one algorithm on technique of random sampling and the other on methods of random permutations. Which one is more efficient?

25. Briefly discuss Importance Sampling.

26. Use Monte Carlo methods to estimate the expected value of fifth order statistic from a sample of size 20 from a $N(0, 1)$ distribution. Estimate the variance of the estimator.

27. Consider the integral $\int_0^{\infty} x^2 \sin(\pi x) e^{-\frac{x}{2}} dx$. Use Monte Carlo method with an exponential weight to estimate the integral.

28. Suppose you are given a biased six faced die, where the probabiltiy of getting any faces 1, 2, .., 5 are equally likely and 6 appears with probability 0.10. Given 4 observations from uniform (0, 1) as, 0.18, 0.9352, 0.1080, 0.0063. Simulate 4 rolls of this die.

Answer ***any two*** questions from question nos. **29-31**. 10×2

29. How do you generate random variates from a truncated $N(0, 1)$ distribution over $(-1, 2)$?

30. Let X_1 and X_2 be iid Cauchy (0, 1) r.v's and let $Y = \alpha X_1 + (1-\alpha) X_2$, where $0 \leq |\alpha| \leq 1$.

Describe an algorithm for generating an observation from the distribution of Y.

31. Let U be uniform $(-1, 1)$. How do you use simulation to approximate the following :

(a) $\mathrm{Corr}\left(U, \sqrt{1-U^2}\right)$

(b) $\mathrm{Corr}\left(U^2, \sqrt{1-U^2}\right)$?